Research Seminars

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Design of Unicyclic Networks

Given a weighted undirected graph, we want to partition the set of vertices into some connected components such that each connected component contains exactly one cycle. The objective function is given by the weight of the edges included in the solution.

First, we prove that this problem is easy to solve. Then, we add a new technical constraint related to the size of cycles: the solution should not contain cycles of length less than a certain bound. This constraint makes the problem difficult.

A polyhedral study is proposed. Many facets and valid inequalities are derived. Some of them can be exactly separated in polynomial time. Hence the network design problem is solved by a cutting plane algorithm based on these inequalities and using a compact formulation derived from the transversality of the bicircular matroid. Computational experiments show that this approach is quite efficient.

We also consider another important variation of the problem. Some given special nodes must belong to cycles. We still want the connected components to be unicyclic while the cycle size constraint is ignored. We show that this problem is a generalization of the perfect binary 2-matching problem. It turns out that the problem is easy to solve. An exact extended linear formulation is provided. We also present a partial description of the convex hull of the incidence vectors of these Steiner networks. Polynomial time separation algorithms are described. One of them is a generalization of the Padberg-Rao algorithm to separate blossom inequalities.

This is a joint work with Hadji.

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Routing Under Uncertainty

Due to the success of the Internet and the diversity of communication applications, it is becoming increasingly difficult to forecast traffic patterns. To capture the traffic variations, a flexible model where traffic belongs to a polytope was introduced in 2001 (Ben-Ameur and Kerivin). Using this uncertainty model, it is possible to compute a robust stable routing which is valid for any traffic matrix inside the polytope. It is also theoretically possible but practically difficult to consider a fully dynamic strategy where routing depends on the current traffic matrix. We propose a strategy that can be seen as a compromise between robust routing and dynamic routing. It consists in partitioning the uncertainty set into some subsets and considering a robust routing for each subset. A theoretical study of this problem is provided in this talk.

This partitioning approach is further improved and generalized by Ben-Ameur and Zotkiewicz (2009-2010) leading to a new routing paradigm.

On the Maximum Cut Problem

The maximum cut problem is a classical combinatorial optimization problem that is known to be NP-hard in general. In the present paper we provide some new lower and upper bounds that are based on the eigenvalues of the weight matrix with modified diagonal entries. Namely, we show that some upper bounds presented here are generally better than the SDP bound introduced by Goemans and Williamson. We also discuss the complexity of computing these bounds and provide some preliminary computational results.

The spectral bounds are more or less based on the solution of some quadratic problems $\min_{x \in \{-1,1\}^n} x^t Qx$ where \$Q\$ is symmetric and has a limited rank. It turns out that this problem is related to a classical combinatorial geometry problem where we want to determine the cells of an arrangement of hyperplanes. We present a new and efficient recursive algorithm to solve the problem. We also prove that solving the problem $\min_{x \in \{-1,1\}^n} x^t Qx$ is easy if the rank of the matrix Q is fixed and the number of its positive diagonal entries is O(log(n)).

Finally, we present a new efficient heuristic for the maximum cut problem exploiting the semi-definite approach of Goemans and Williamson but without solving any semidefinite program.

This is a joint work with Neto.

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Multiple Point Separation and In-Out Algorithm

In order to solve linear programs with a large number of constraints, constraint generation techniques are often used. In these algorithms, a relaxation of the formulation containing only a subset of the constraints is first solved. Then a separation procedure is called which adds to the relaxation any inequality of the formulation that is violated by the current solution. The process is iterated until no violated inequality can be found. In this talk, we present a separation procedure that uses several points to generate violated constraints.

The complexity of this separation procedure and of some related problems is studied.

We also describe a second approach to accelerate both cutting plane and column generation algorithms. We show that we can achieve this goal by choosing good separation points. Focus is given on problems for which we have an exact separation oracle. An In-Out algorithm is proposed and the convergence is proved under some general assumptions. The separation point is a convex combination of the best known interior point and the optimum of the current relaxation. Computational experiments related to three classes of problems, survivable network design, multicommodity flow problems and random linear programs, clearly point out the savings of time allowed by the simple In-Out approach proposed in this talk.

This is a joint work with Neto.

Some Generalizations of the Minimum Cut Problem

Given G=(V,E) an undirected graph and two specified nonadjacent nodes a and b of V, a cut separator is a subset $F=\delta(C)\subseteq E$ such that $a,b\in V-C$ and a and b belong to different connected components of the graph induced by $V-C$. Given a nonnegative cost vector $c\in IR_+^{|E|}$, the optimal cut separator problem is to find a cut separator of minimum cost. This new problem is closely related to the vertex separator problem. In this talk, we give a polynomial time algorithm for this problem. We also present six equivalent linear formulations, and we show their tightness. Using these results we obtain an explicit short polyhedral description of the dominant of the cut separator polytope.

Other generalizations of the minimum cut problem will also be presented. This is a joint work with Didi-Biha.

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Some problems related to constrained length connectivity

Some problems related to length connectivity are addressed in this talk. Let $S_l(x,y)$ be the minimum number of vertices that should be removed to destroy all the paths of length at most l between two vertices x and y. Let $I_l(x,y)$ be the maximum number of such node-disjoint paths. We first focus on f(l,d) defined as the supremum of $\frac{S_l(x,y)}{I_l(x,y)}$ taken over all graphs and all pairs of x, y separated by a distance d. One of the results shown in this paper states that this supremum is exactly equal to l + 1 -d when $d \geq \left\lceil \frac{2}{3}(l+1) \right\rceil$ and is constant when $2 \leq d \leq 2 + \left\lfloor \frac{l+1}{3} \right\rfloor$.

Relationships between flows and constrained length connectivity are addressed. We give both extended and compact formulations to compute $S_l(x,y)$, $I_l(x,y)$ and $F_l(x,y)$ (the maximum fractional flow that can be sent from x to y through hop-constrained paths).

Some classes of two connected graphs satisfying path length constraints are defined. Most of them describe survivable telecommunication networks.

We also study the minimum edge numbers of these two connected graphs. Some of their topological properties are presented.

A part of this work is done with Luis Gouveia.

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