LECTURE I

THE GEOMETRY OF SINGULARITIES

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Singularity theory is concerned with the geometry and topology of spaces and mappings which are not regular.

The field was started by a series of papers of Hassler Whitney in the 1940s and 1950s. In these classical papers the notion of stable mappings was formulated. Whitney's work was followed up by the work of René Thom in the 1960s, who introduced the concept of versal unfolding, perhaps the greatest contribution of singularity theory to sciences. Building on Whitney and Thom's ideas, John Mather, in a remarkable series of papers, from 1968 to 1971, characterized stability in terms of transversality, calculated the "nice range of dimensions" for which stable maps are dense, and made considerable progress on the classification of germs of stable maps.

In this talk, we shall review classical results of singularity theory, and show how to apply them to study the geometry of curves and surfaces.

The following topics will be discussed:
- Stable mappings;
- Thom's classification of elementary catastrophes.
- Arnol'd classification of simple singularities;
- Examples.