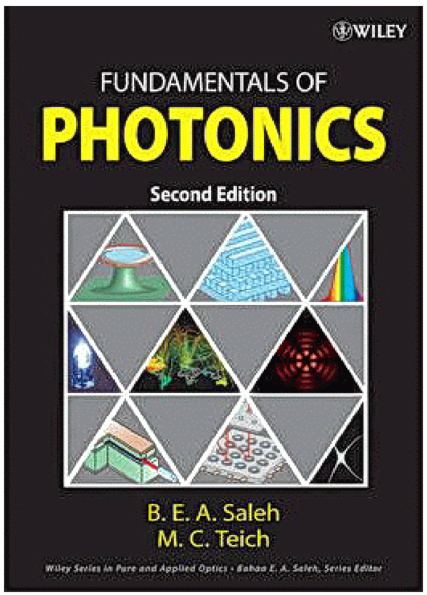
WARSAW UNIVERSITY OF TECHNOLOGY
DEVELOPMENT PROGRAMME

Quantum Electronics
Lecture 2



Waveguide optics & Devices

Lecturer:

Bozena Jaskorzynska

Royal Institute of Technology (KTH)
Sweden, bj@kth.se

HUMAN CAPITAL
NATIONAL COHESION STRATEGY

EUROPEAN UNION
EUROPEAN
SOCIAL FUND

Lectures co-financed by the European Union in scope of the European Social Fund

Contents

♦ Guiding in slab waveguides

♦ Effective Index Method

♦ Coupled Mode Theory

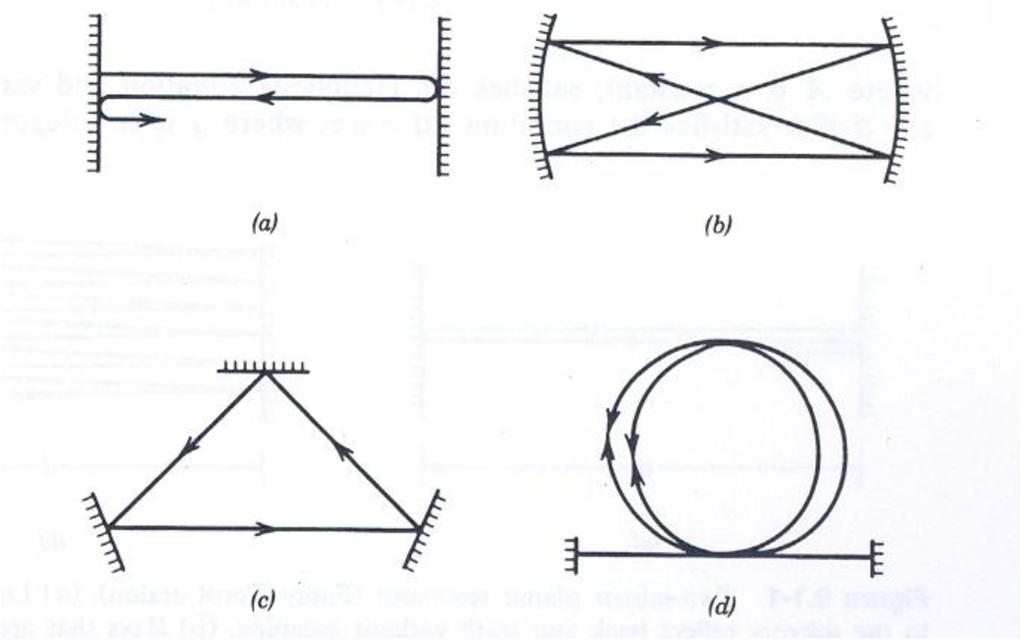
♦ Mode interference

♦ Examples of device concepts

Saleh Ch. 8

What is an optical resonator ?

Optical resonator is a trap for light !!



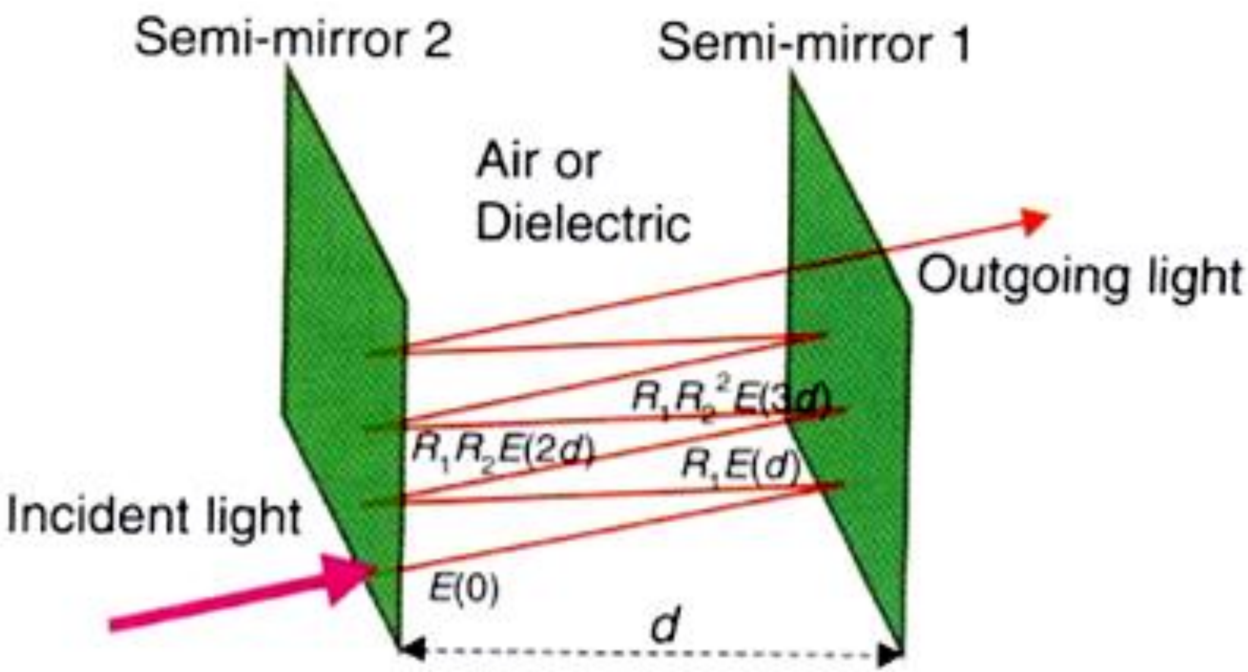
Confines and stores light at certain resonance frequencies

Light circulates or is repeatedly reflected

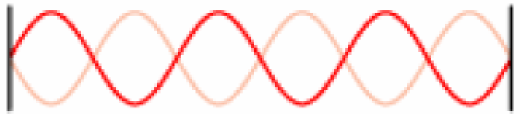
High frequency selectivity

Applications:
"Container" for laser light
Optical filter or spectrum analyzer

Principle for Fabry Perot resonator



The outgoing λs for which d = m λ/2, add up in phase (resonant λs)



Condition for standing wave in a resonator

Fabry-Perot resonator

$$E_0 \propto e^{inkz} \qquad E_1 \propto e^{i(nkz+nk2d)}$$

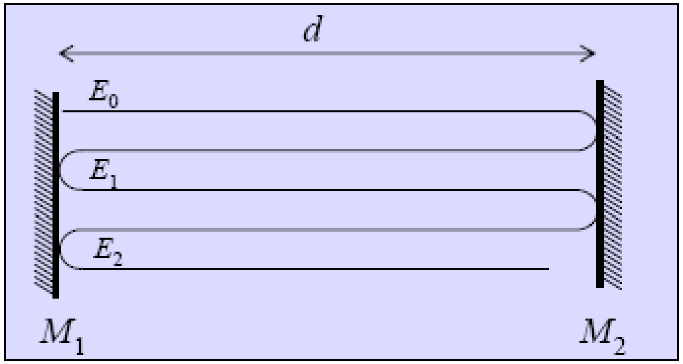
Phase shift per round trip (ignoring phase shifts on reflection)

$$2\theta = 2kd\,\boldsymbol{n}$$

Resonans condition

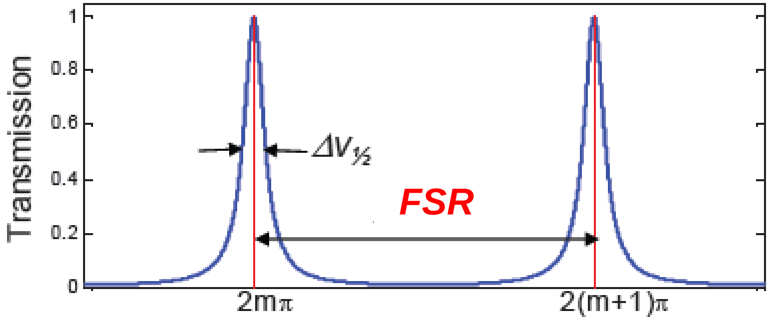
If this is $m2\pi$, then the E_i's add coherently, i.e. we are on resonance

(m is an integer)



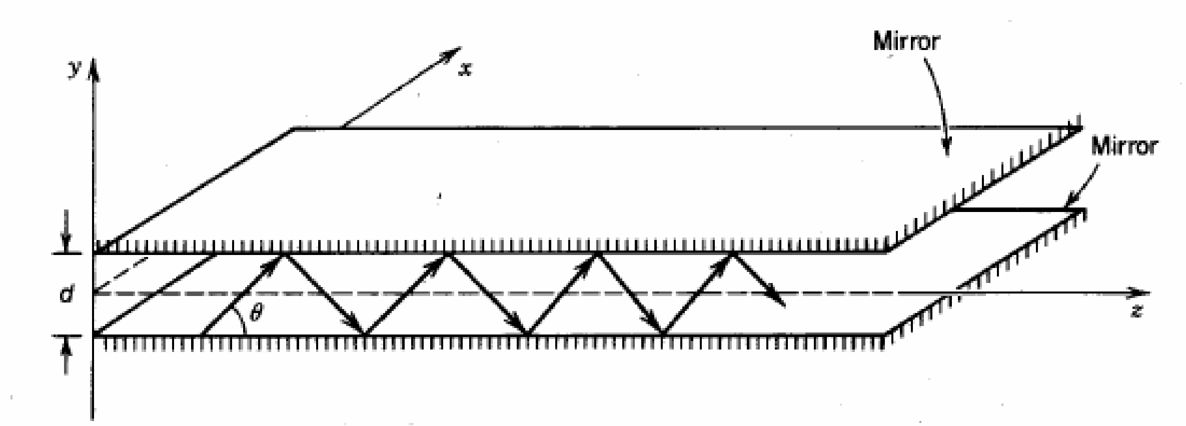
Spacing of resonances is called the Free Spectral Range (FSR)

$$\boxed{FSR = \frac{c}{2d\,\boldsymbol{n}}}$$



Planar-mirror (”closed”) waveguides

Let us use this simplest case to explain basic concepts for waveguiding



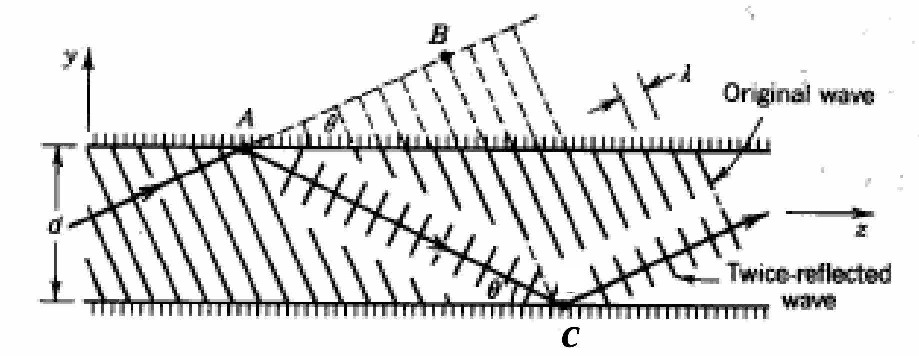
Imagine a monochromatic plane wave bouncing between two parallel, perfectly reflecting metal mirrors
The field is captured and in such a way can be guided

It CAN but IS it ??

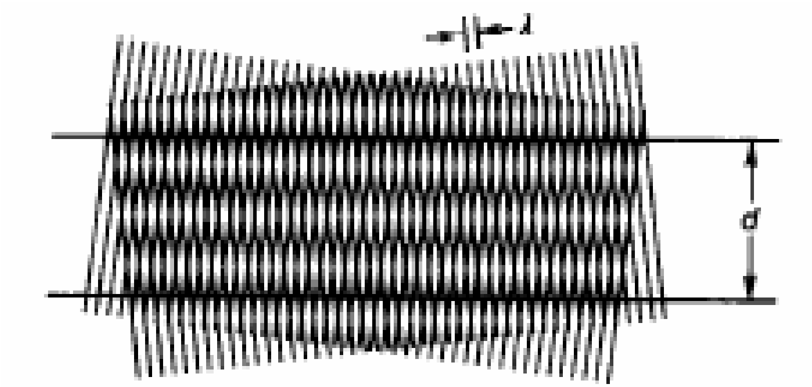
Fundamentals of Photonics - Saleh and Teich

Self-consistency creates modes

Guidance can only occur at angles at which self-consistency (transverse resonance) condition is satisfied: as a wave reflects twice it duplicates itself

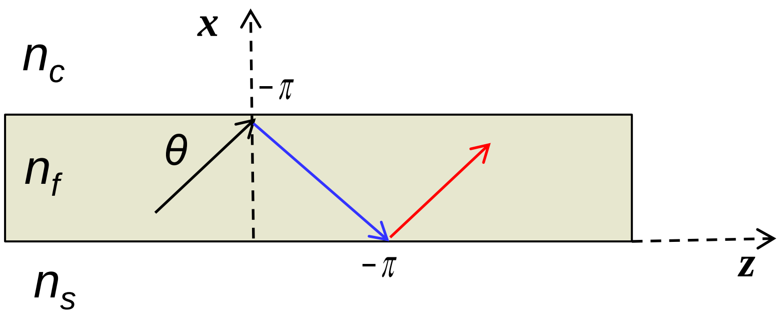


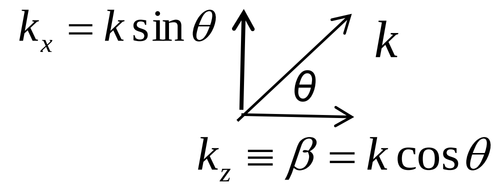
At those angles the two waves interfere to create a pattern that dose not change with Z (forming a transverse field distribution, "profile" , of a guided mode)



Fundamentals of Photonics - Saleh and Teich

Self-consistency condition





$$E_0 \propto e^{-i(k_x x + k_z z)}$$

$$E_1 \propto e^{i\pi} e^{-i(k_x x + k_z z) - ik_x 2d} e^{i\pi} \Longrightarrow \varphi_1 - \varphi_0 = k_x 2d - 2\pi$$

Transverse resonance condition:

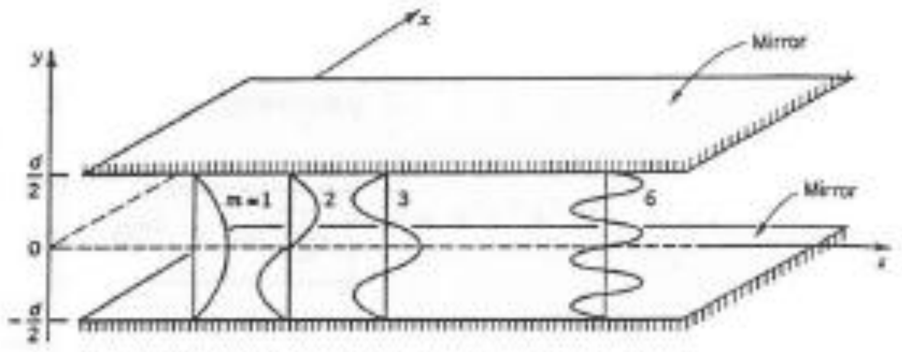
$$2kd \sin\theta_m - 2\pi = m \cdot 2\pi \qquad m = 0, 1, 2, \ldots.$$

Mode order limited by: $\sin\Theta_m \leq 1$

$$\beta_m^2 = k^2 - \frac{m^2\pi^2}{d^2} \Longrightarrow \beta_m^2 = \frac{n^2\omega^2}{c^2} - \frac{m^2\pi^2}{d^2}$$

Dispersion relation

Modal fields in a planar-mirror waveguide



Found from Helmholtz equation
+ boundary conditions at the walls
(Boundary-Value problem)

At the walls tangential components of E and H must be continuous

(For metal mirrors E at the walls = 0)

$$E_x(y,z) = a_m u_m(y)\exp(-j\beta_m z),$$

$$u_m(y) = \begin{cases} \sqrt{\dfrac{2}{d}}\cos\dfrac{m\pi y}{d}, & m = 1,3,5,\ldots \\ \sqrt{\dfrac{2}{d}}\sin\dfrac{m\pi y}{d}, & m = 2,4,6,\ldots, \end{cases}$$

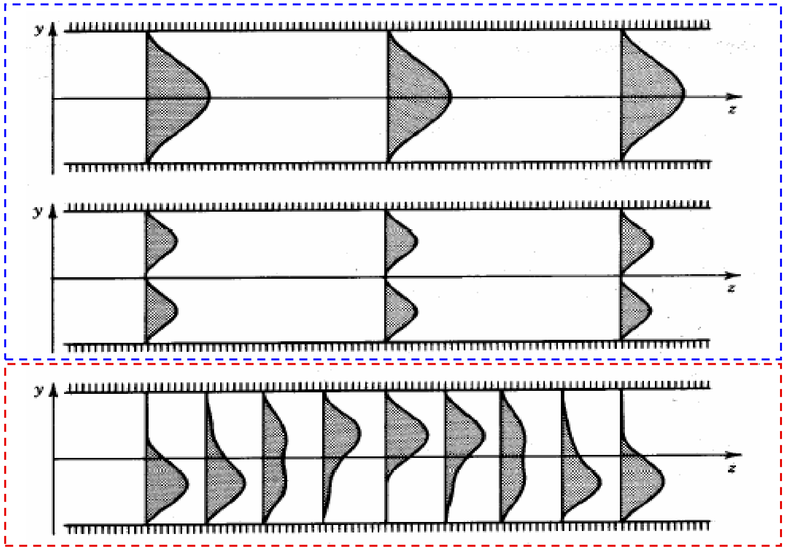
Orthogonality: $\int_{-d/2}^{d/2} u_m(y)u_l(y)\,dy = 0, \qquad l \neq m,$

Normalization: $\int_{-d/2}^{d/2} u_m^2(y)\,dy = 1$

Orthogonality* – each mode carries its own power and *does not interact with the others

Fundamentals of Photonics - Saleh and Teich

Single-mode and "mixed" propagation



Beat length L_B

Intensity for single mode propagation ***does not vary*** *with distance z*

Intensity for several modes propagating together ***varies*** *with* ***z*** *since they interfere with the* ***relative phase*** *which* ***varies*** *along* ***z***

$$E(y,z) = u_1(y)e^{-i\beta_1 z} + u_2(y)e^{-i\beta_2 z}$$

$$I \propto |E|^2 = |u_1|^2 + |u_2|^2 + \left[u_1 u_2^* e^{-i(\beta_1-\beta_2)z} + cc\right]$$

$$= I_2 + I_2 + 2u_1 u_2 \cos(\beta_1 - \beta_2)z \quad \textit{(for real u)}$$

$$\boxed{L_B = \frac{2\pi}{\beta_1 - \beta_2}}$$

But there is NO power power exchange between the modes:

$$P \propto \int |E|^2 dy = \int |u_1|^2 dy + \int |u_2|^2 dy + \left[e^{-i(\beta_1-\beta_2)z} \boxed{\int u_1 u_2^* dy} + cc \right] = P_1 + P_2$$

=0 due to orthogonality

Dielectric (“open”) waveguides

Light guiding by total internal reflection (TIR)

Discovered by Daniel Colladon in 1841 in water jet

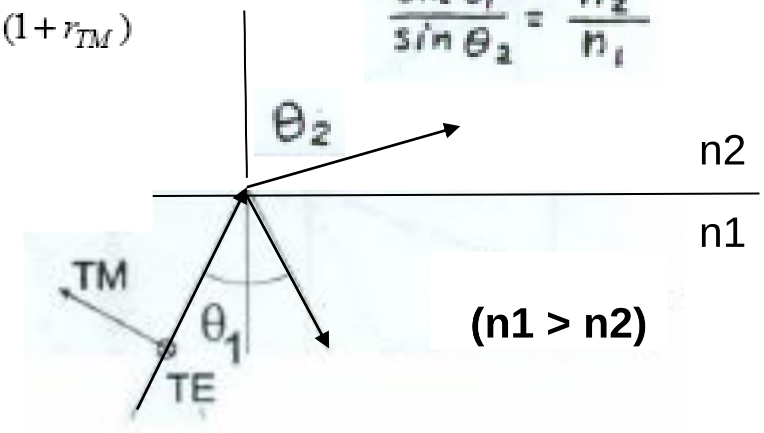
Total internal reflection

For TE wave: $r_{TE} = \dfrac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$ $\qquad t_{TE} = 1 + r_{TE}$

For TM wave: $r_{TM} = \dfrac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$ $\qquad t_{TM} = \dfrac{n_1}{n_2}(1 + r_{TM})$

$$r_{TE} = |r_{TE}| \exp(j\phi_{TE}), \quad r_{TM} = |r_{TM}| \exp(j\phi_{TM})$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$



At optical wavelengths one uses dielectric (open) waveguides where the light is confined due to the ***total internal reflection****:* $n_1 \cdot \sin\theta_1 \geq n_2$

$$\Rightarrow \quad r_{TE} = \exp(j\phi_{TE}), \quad r_{TM} = \exp(j\phi_{TM})$$

Phase shift at the total internal reflection

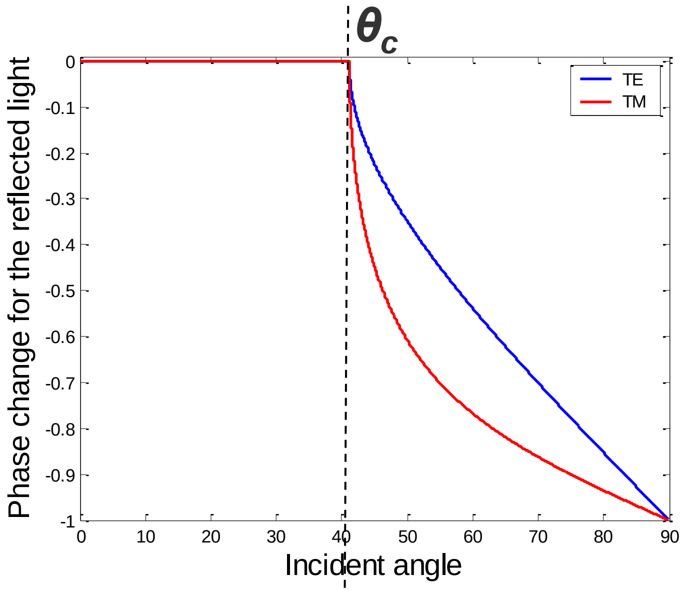
Phase shift depends on the incidence angle θ_1

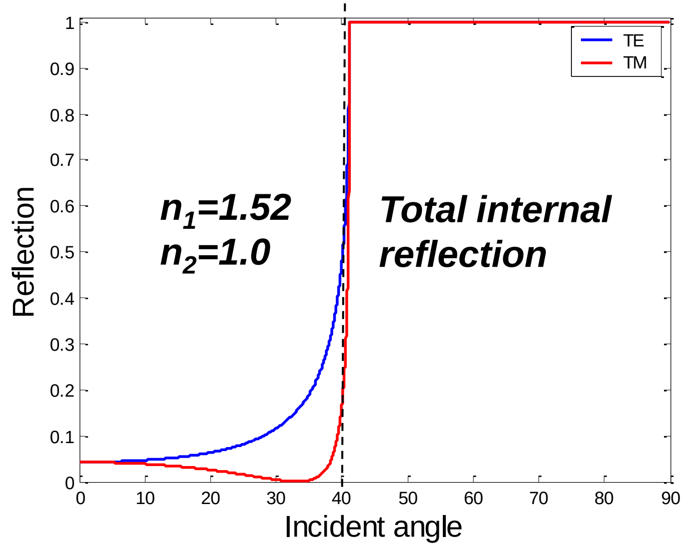
$$\tan\frac{\phi_{TE}}{2} = \frac{\sqrt{\sin^2\theta_1 - \sin^2\theta_c}}{\cos\theta_1} = \frac{\sqrt{n_1^2\sin^2\theta_1 - n_2^2}}{n_1\cos\theta_1}$$

$$\tan\frac{\phi_{TM}}{2} = \frac{\sqrt{\sin^2\theta_1 - \sin^2\theta_c}}{\cos\theta_1\sin^2\theta_c} = \frac{n_1^2}{n_2^2}\frac{\sqrt{n_1^2\sin^2\theta_1 - n_2^2}}{n_1\cos\theta_1}$$

θ_c *- critical angle* $\quad \sin\theta_c = \frac{n_2}{n_1}$

The phase shift for Φ drops from 0 to -π

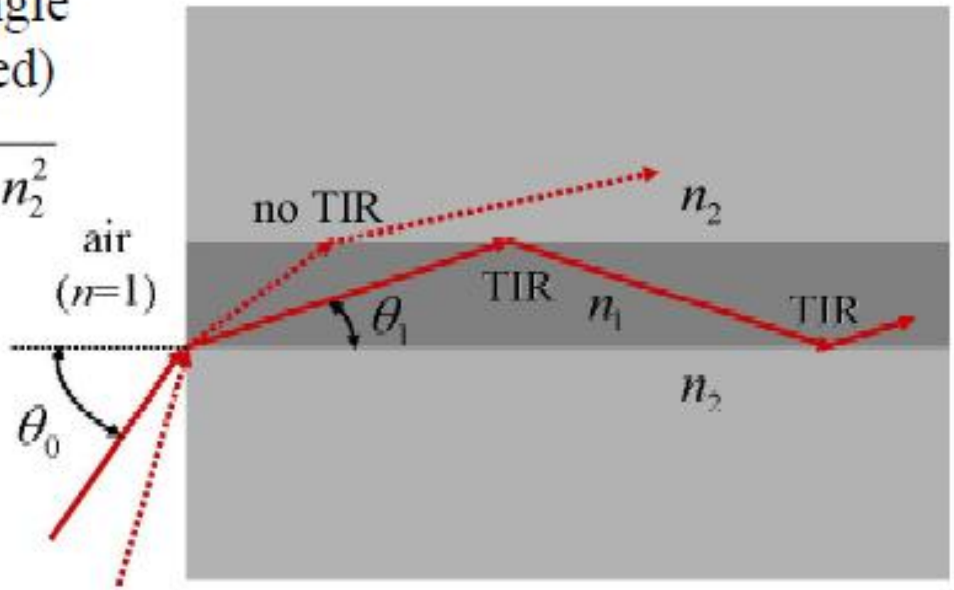




Numerical aperture NA

NA – sin(largest angle that is waveguided)

$$NA = \sin\theta_0 \leq \sqrt{n_1^2 - n_2^2}$$



high index contrast ($n1/n2$) $\longrightarrow$ high NA

Important for light incoupling to a waveguide !!

Transverse resonance (consistency) condition

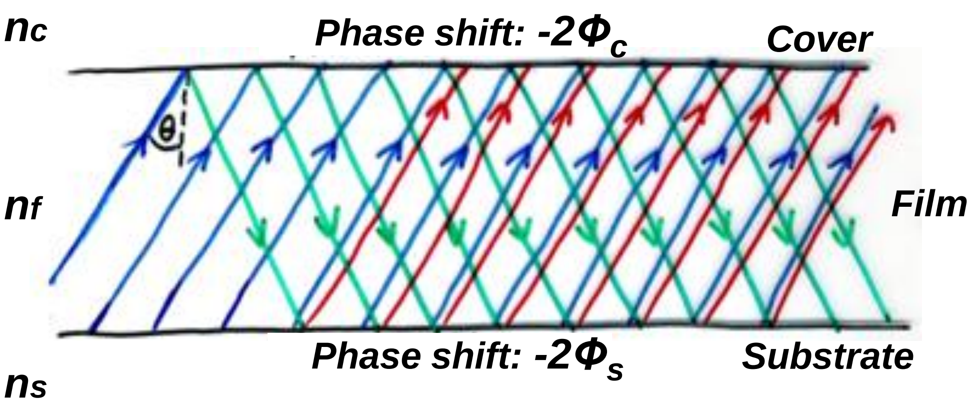
Light will not escape from a slab (film) when:

$$\boxed{n_c < N < n_f}$$ *where:* $N \equiv \beta/k = n_f \sin\theta$ ← **Effective index**

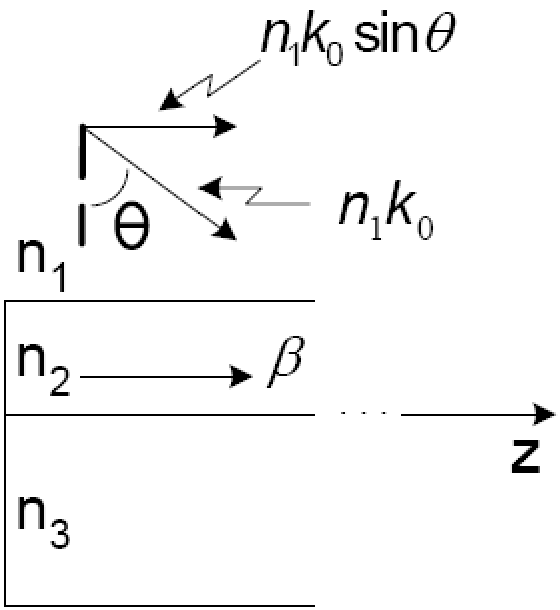
But this is not sufficient for light guidance !

In addition the transverse resonance (self-consistency) condition must be satisfied - the incident and the doubly reflected wave must be in phase:

$$n_f > n_s > n_c$$



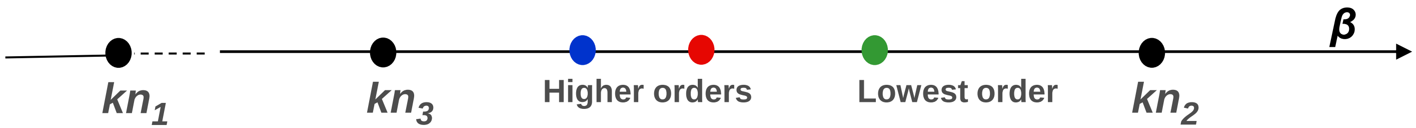
Side coupling to a waveguide



Not possible to match $k_{1z} = k_{2z}$ ***since:***

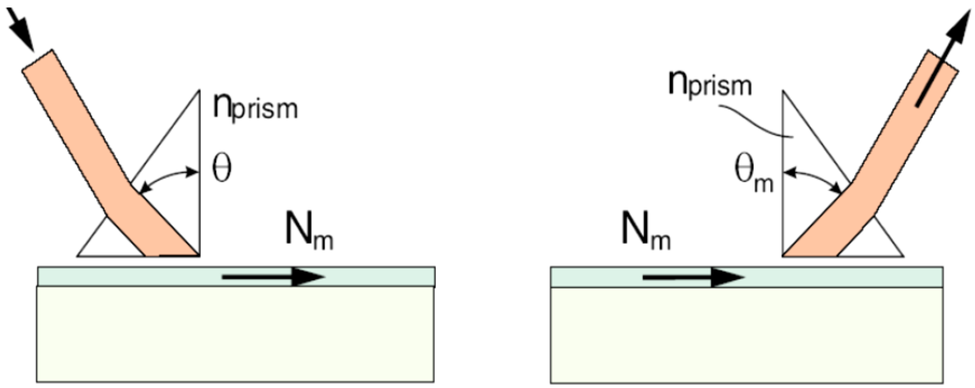
$$\boxed{\beta > n_1 k_0}$$

Not possible to couple light directly from the side



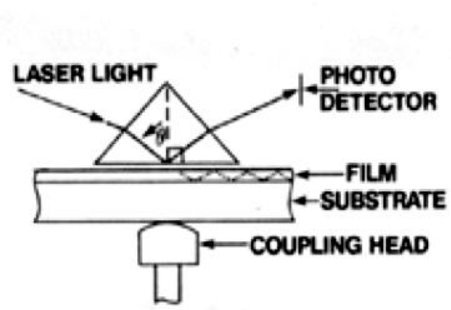
Possible with a prism of refractive index ≥ n_2

Prism coupling

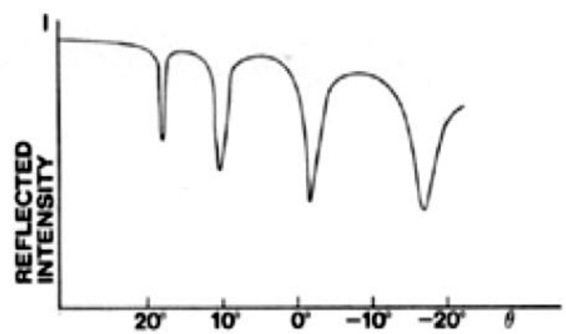


*Coupling to mode **m** if:* $n_{prism} k_0 \sin(\theta) = \beta_m = n_2 k_0 \sin(\theta_m) = k_0 N_m$

Experimental determination of mode propagation constants $\beta_m = n_2 k_0 \sin(\theta_m)$

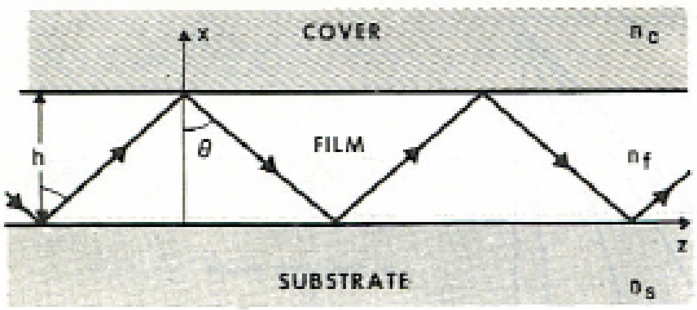


Prism coupler



Intensity of reflected ligth vs angle of incididence

Dispersion relation



Phase shift for TE (i = s or c):

$$\phi_i = 2\tan^{-1}\left(\frac{n_f^2\sin^2\theta - n_i^2}{n_f^2\cos^2\theta}\right)^{1/2} = 2\tan^{-1}\left(\frac{N^2 - n_i^2}{n_f^2 - N^2}\right)^{1/2}$$

Transverse resonance condition:

$$\boxed{2kn_f h\cos\theta - 2\phi_c - 2\phi_s = 2m\pi}$$

m : mode number

$kn_f h\cos\theta$: phase shift for the transverse passage through the film

$2\phi_c (= \phi_{TE,TM})$: phase shift due to total internal reflection from film/cover interface

$2\phi_s (= \phi_{TE,TM})$: phase shift due to total internal reflection from film/substrate interface

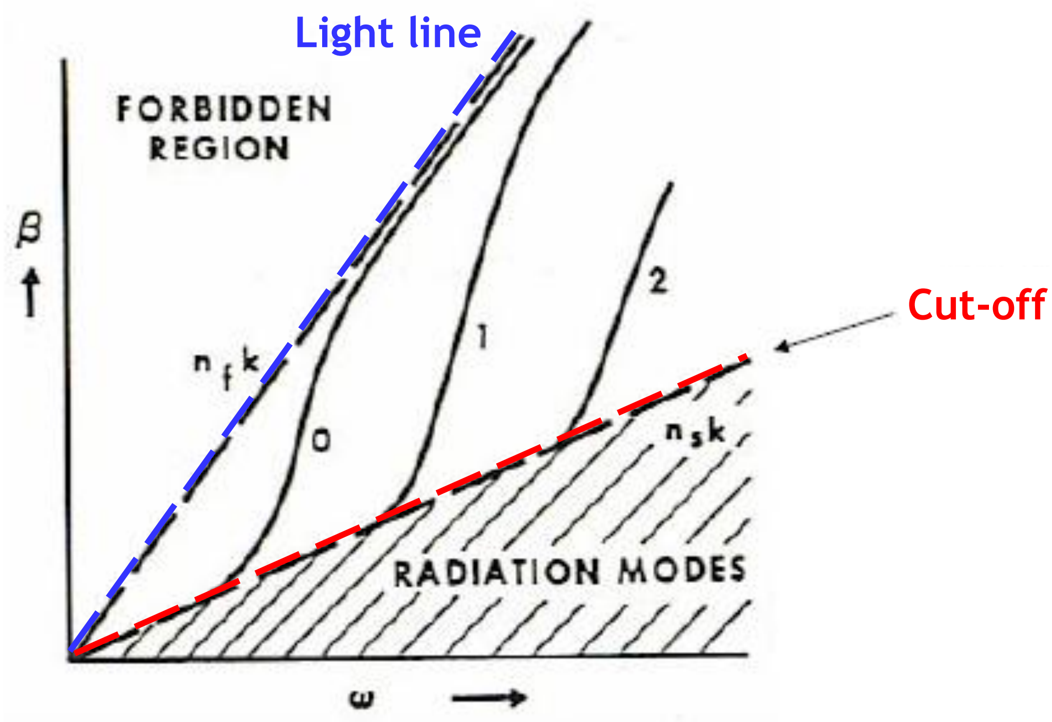
Dispersion equation (β vs. ω):

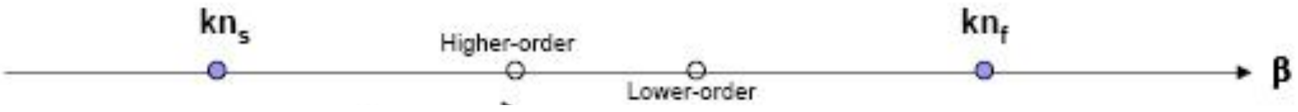
$$\boxed{kn_f h\cos\theta - \phi_c - \phi_s = m\pi}$$

Effective index $N \equiv \beta/k = n_f\sin\theta$

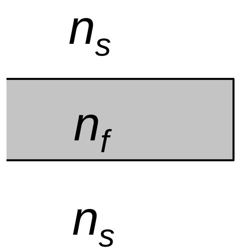
$\boxed{n_s < N < n_f}$ **For guided modes**

Typical dispersion diagram

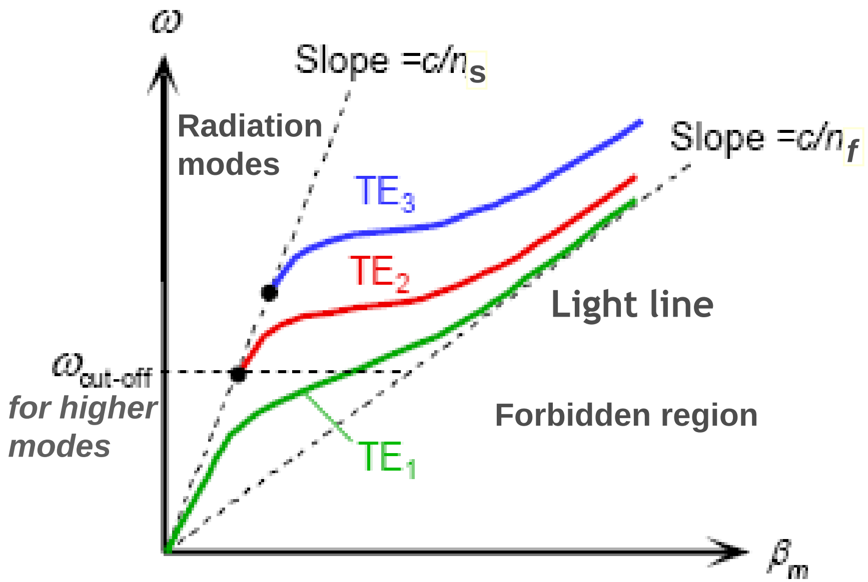




Symmetric waveguide

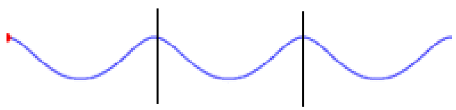


Group and effective indices in waveguides



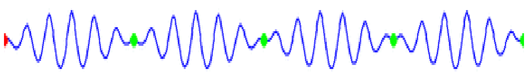
Phase velocity v_p : speed of wave fronts

Can be larger than c !



$$v_p = \frac{\omega}{\beta}$$

Group velocity v_g: velocity of wave packets (information)

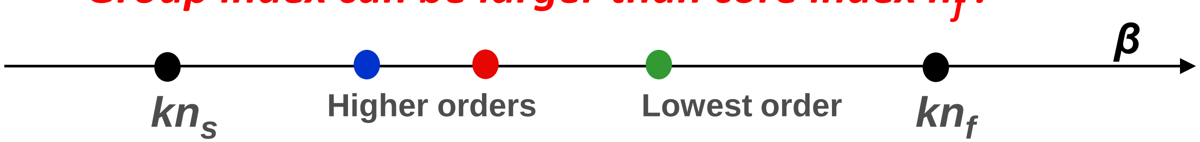


$$v_g = \frac{d\omega}{d\beta}$$

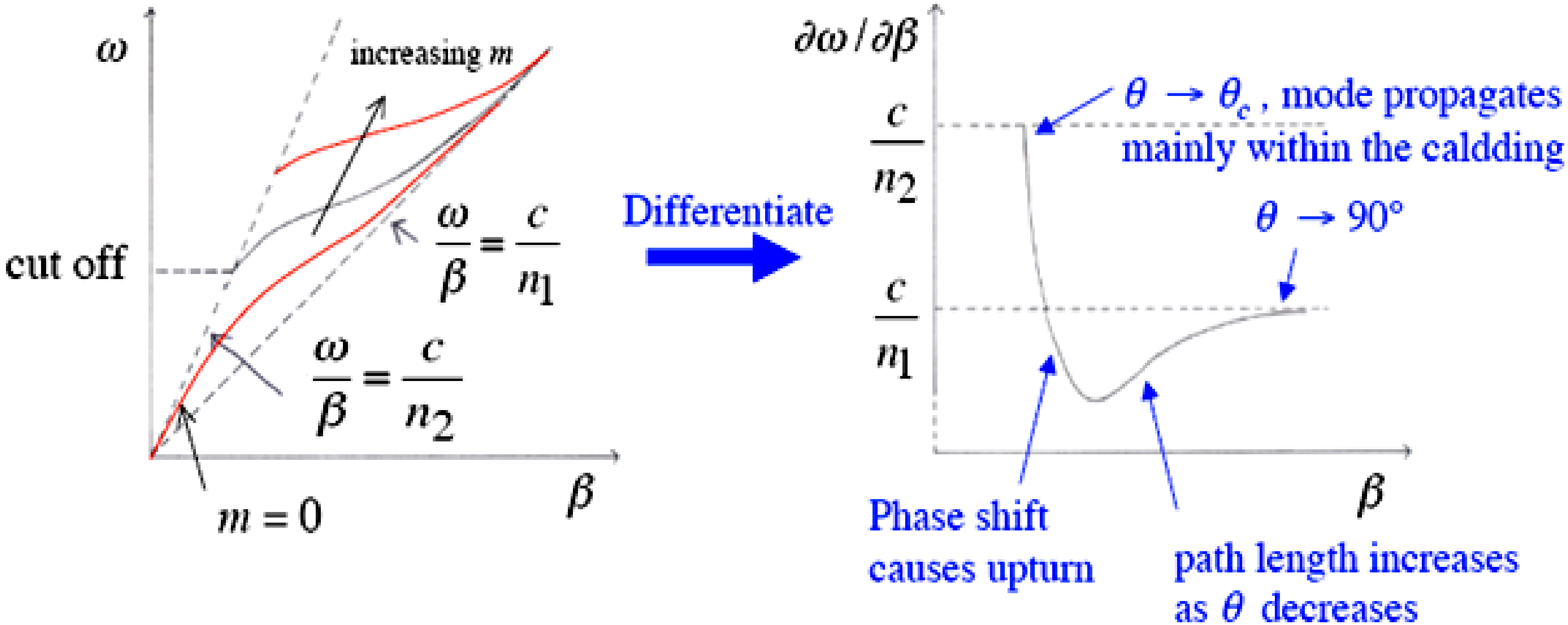
Group index: $n_g = \frac{c}{v_g} = c\frac{d\beta}{d\omega}$

Effective index: $N_m = \frac{c}{v_p} = c\,\frac{\beta_m}{\omega}$

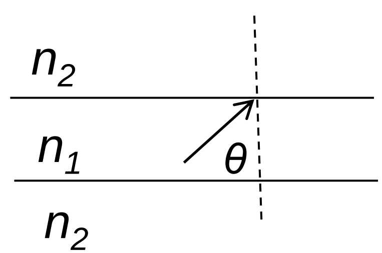
- ***Effective index is always smaller than core index***
- ***Group index can be larger than core index n_f !***



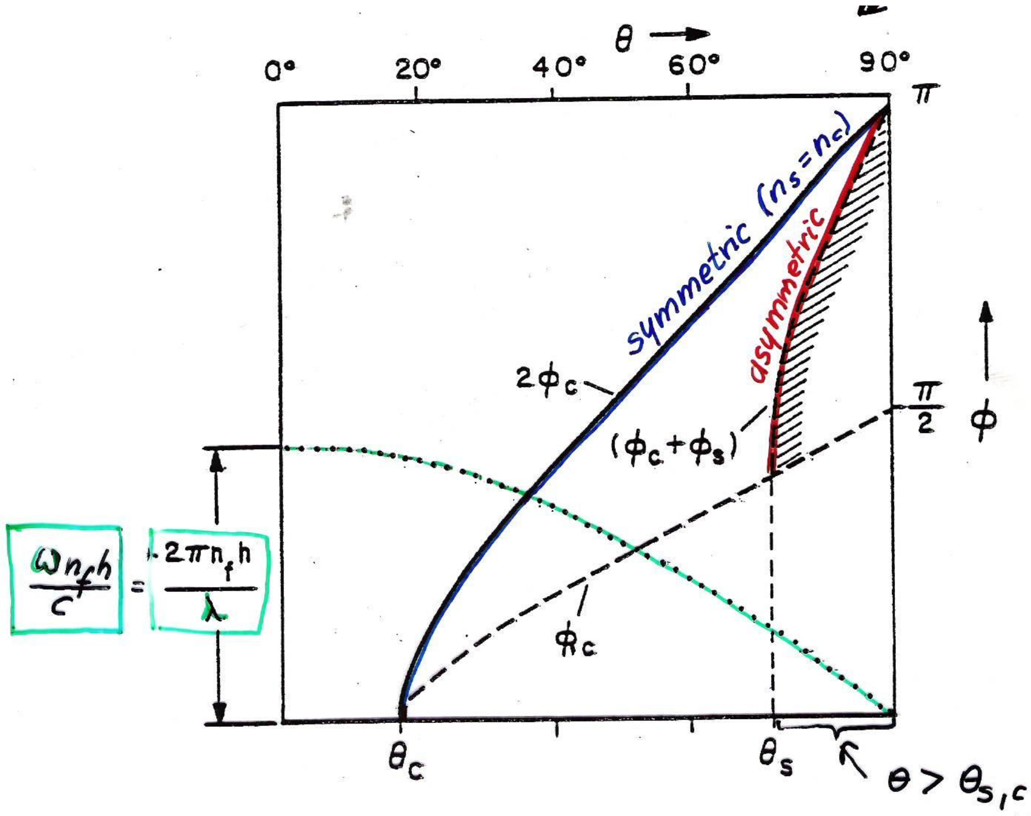
Group velocity for guided modes

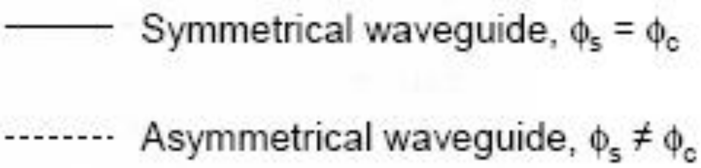


Symmetric waveguide



Graphical solution of the dispersion equation





$(2\pi\, n_f\, h \cos\theta)/\lambda = \Phi_s(\theta) + \Phi_c(\theta)$

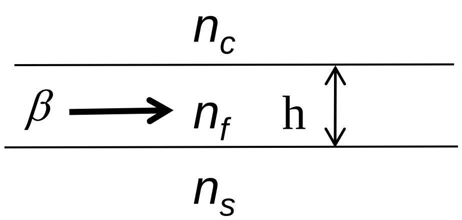
For fundamental mode (m = 0), there is always a solution (no cut-off) for symmetrical waveguide.

Increasing h (and/or decreasing λ) will support more modes.

Normalized units for slab waveguides

Normalized frequency and film thickness

$$V \equiv kh\sqrt{n_f^2 - n_s^2}$$



Normalized guide index

$$b \equiv \frac{N^2 - n_s^2}{n_f^2 - n_s^2} \qquad N \equiv \beta / k$$

b = 0 at cut - off (N = n_s), and approaches 1 as N → n_f.

Measure for the asymmetry

$$a \equiv \frac{n_s^2 - n_c^2}{n_f^2 - n_s^2} \text{ for TE,} \qquad a \equiv \frac{n_f^4}{n_c^4}\frac{n_s^2 - n_c^2}{n_f^2 - n_s^2} \text{ for TM}$$

Waveguide	n_s	n_f	n_c	a_E	a_M
GaAlAs, double heterostructure	3.55	3.6	3.55	0	0
Sputtered glass	1.515	1.62	1	3.9	27.1
Ti-diffused $LiNbO_3$	2.214	2.234	1	43.9	1093
Outdiffused $LiNbO_3$	2.214	2.215	1	881	21206

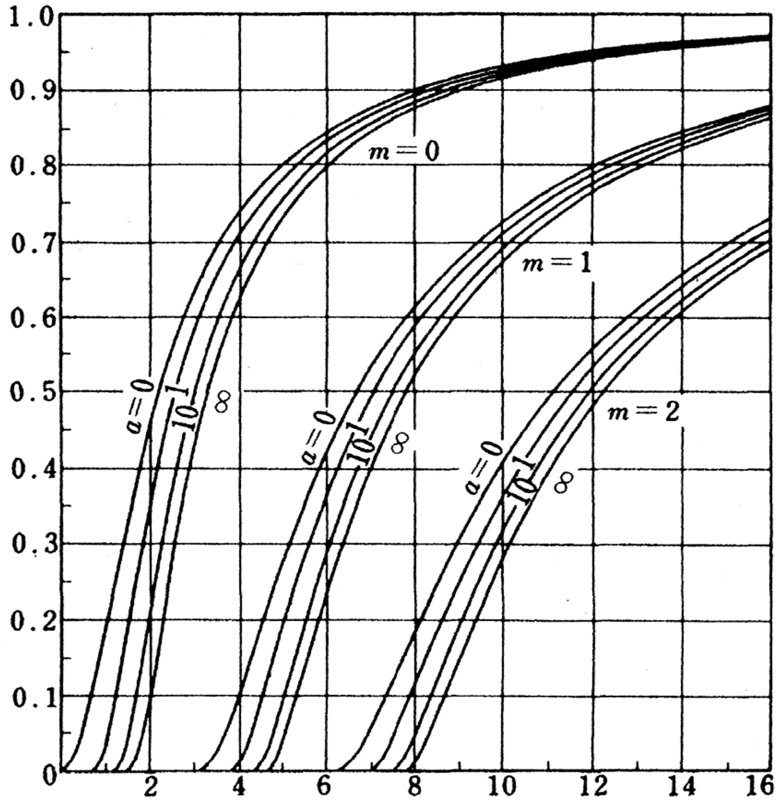
Normalized dispersion diagram

For TE modes, dispersion relation

$$k n_f h\cos\theta - \phi_c - \phi_s = m\pi \rightarrow$$

$$V\sqrt{1-b} = m\pi + \tan^{-1}\sqrt{\frac{b}{1-b}} + \tan^{-1}\sqrt{\frac{b+a}{1-b}}$$

b



$$V \equiv kh\sqrt{n_f^2 - n_s^2}$$

m : Mode number

(Normalized) cut-off frequency:

$$V_0 = \tan^{-1}\sqrt{a}$$ **(for fundamental mode)**

$$V_m = V_0 + m\pi$$

\# of guided modes allowed:

$$m = \frac{2h}{\lambda}\sqrt{n_f^2 - n_s^2}$$

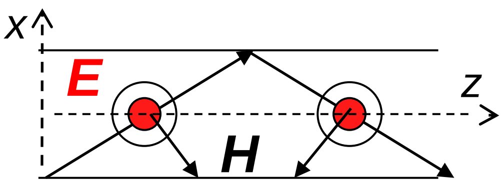
From the normalized dispersion diagram one can find modal propagation constants.

To find modal field distributions (profiles) one has to solve Helmholtz equations with appropriate boundary conditions.

Modal profiles

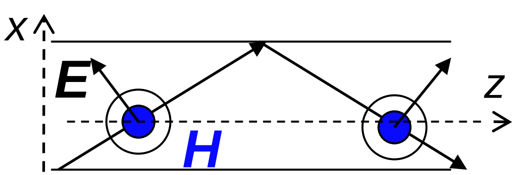
To find modal field distributions (profiles) one has to solve Helmhotz equations - in general, coupled (vectorial problem)!

For a slab waveguide they decouple into two scalar problems for TE and TM polarizations



TE: $Hy = Ex = Ez = 0$, *other components expressed by one scalar, e.g. Ey*

$$H_x = \frac{i}{\omega\mu}\frac{\partial E_y}{\partial z},\ H_z = -\frac{i}{\omega\mu}\frac{\partial E_y}{\partial x}$$



TM: $Ey = Hx = Hz = 0$, *other components expressed by one scalar, e.g. Hy*

$$E_x = \frac{i}{\omega\mu}\frac{\partial H_y}{\partial z},\ E_z = -\frac{i}{\omega\mu}\frac{\partial H_y}{\partial x}$$

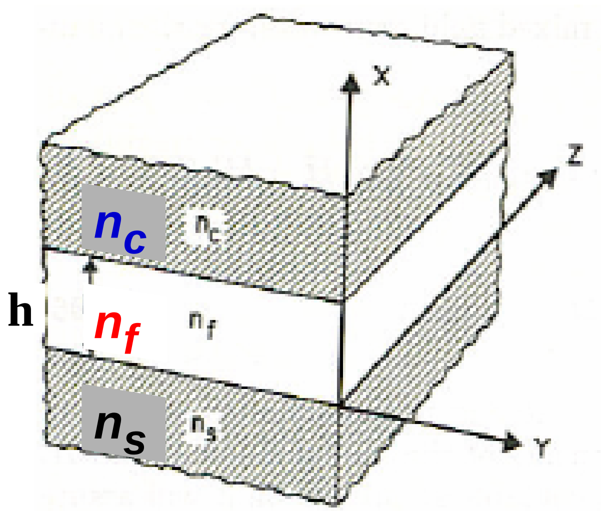
$$\boxed{\frac{\partial^2\Psi}{\partial z^2} + \frac{\partial^2\Psi}{\partial x^2} + k_0^2 n^2(x)\Psi = 0}$$

$$\Psi = \begin{cases} \boxed{Ey} & \textit{for}\ \textbf{TE} \\ \boxed{Hy} & \textit{for}\ \textbf{TM} \end{cases}$$

Look for modal solutions: $\Psi_m(x,z,t) = u_m(x)e^{-i\beta_m z + i\omega t}$

$$\boxed{[\frac{d^2}{dx^2} + (k^2 - \beta_m{}^2)]u_m(x) = 0}$$

Helmholtz equations for modes



For TE: $u_m \equiv E_y \equiv E$

Define:

$$\kappa_c^2 = n_c^2 k^2 - \beta^2 = -\gamma_c^2$$
$$\kappa_f^2 = n_f^2 k^2 - \beta^2$$
$$\kappa_s^2 = n_s^2 k^2 - \beta^2 = -\gamma_s^2$$

Cover: $\frac{\partial^2}{\partial x^2} E(x,y) + (n_c^2 k^2 - \beta^2) E(x,y) = 0 \longrightarrow \frac{\partial^2}{\partial x^2} E - \gamma_c^2 E = 0$

Film: $\frac{\partial^2}{\partial x^2} E(x,y) + (n_f^2 k^2 - \beta^2) E(x,y) = 0 \longrightarrow \frac{\partial^2}{\partial x^2} E + \kappa_f^2 E = 0$

Substrate: $\frac{\partial^2}{\partial x^2} E(x,y) + (n_s^2 k^2 - \beta^2) E(x,y) = 0 \longrightarrow \frac{\partial^2}{\partial x^2} E - \gamma_s^2 E = 0$

Modal field form

Modal solutions are sinusoidal or exponential, depending on the sign of $(k^2 n_i^2 - \beta^2)$

Boundary conditions: The tangential components of E and H are continuous at the interface between layers. $\rightarrow$ E_y and $\partial E_y / \partial x$ continuous at the interface.

For guided modes:

Evanescent field

Cover: $\dfrac{\partial^2}{\partial x^2} E_y - \gamma_c^2 E_y = 0 \quad \longrightarrow \quad E_y = E_c \exp[-\gamma_c (x - h)]$

Film: $\dfrac{\partial^2}{\partial x^2} E_y + \kappa_f^2 E_y = 0 \quad \longrightarrow \quad E_y = E_f \cos(\kappa_f x - \phi_s)$

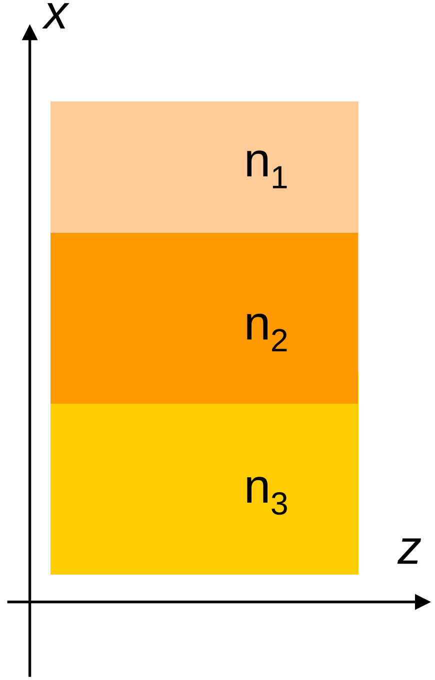
Substrate: $\dfrac{\partial^2}{\partial x^2} E_y - \gamma_s^2 E_y = 0 \quad \longrightarrow \quad E_y = E_c \exp(\gamma_s x)$

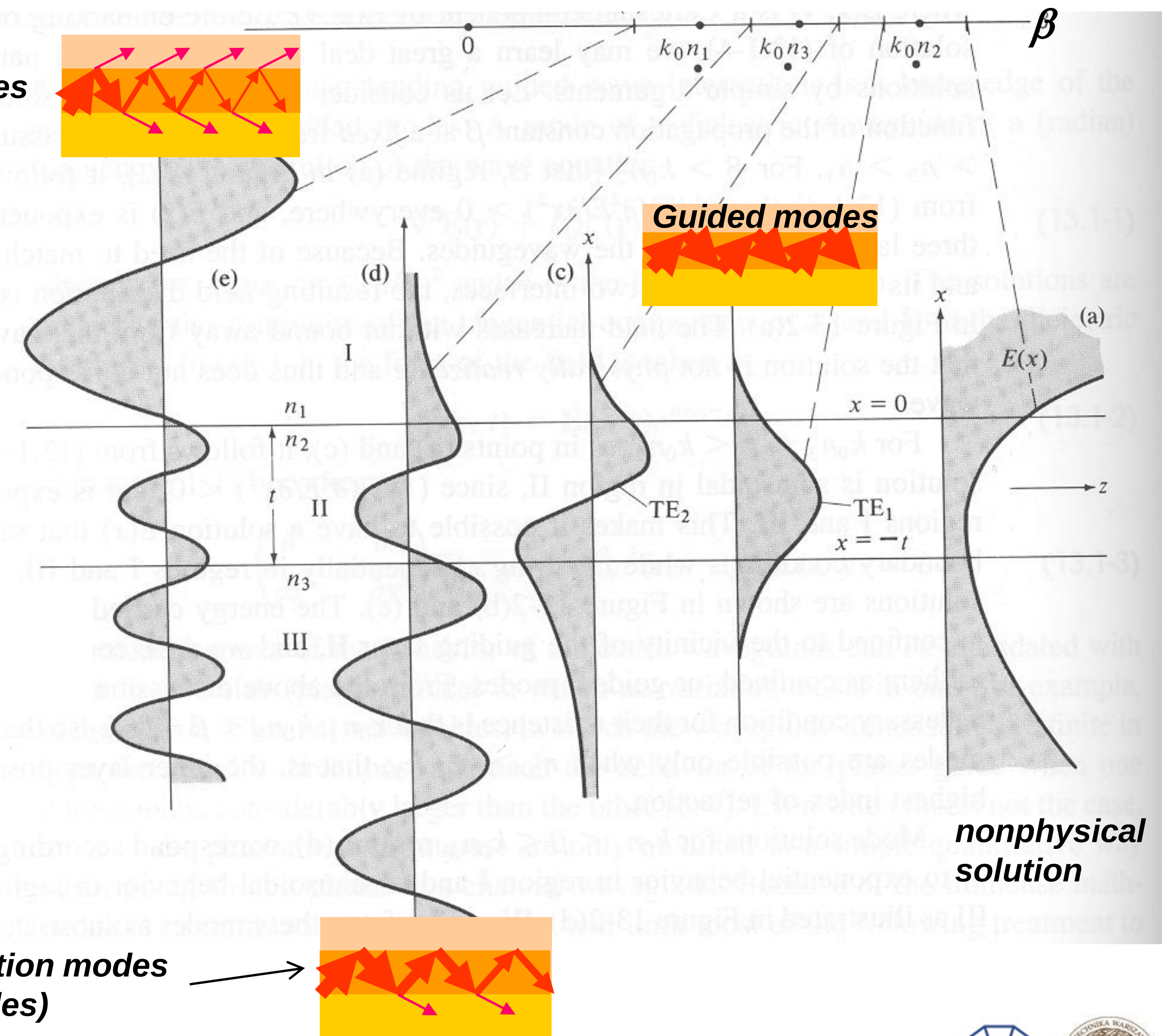
Evanescent field

For solutions satisfying the bondary condition see e.g. Yariv Chapt. 3.1

Types of modes

Radiation modes





nonphysical solution

Substrate radiation modes (also leaky modes)

Waveguide mode general properties

Modes are ORTOGONAL

$$\int_{-\infty}^{+\infty} E_y^{(l)} E_y^{(m)} dx = \frac{2\omega\mu}{\beta_m}\delta_{l,m}$$

Each of them carries its own power and does not interact with the others

Set of all modes (including radiation and evanescent ones) is COMPLETE

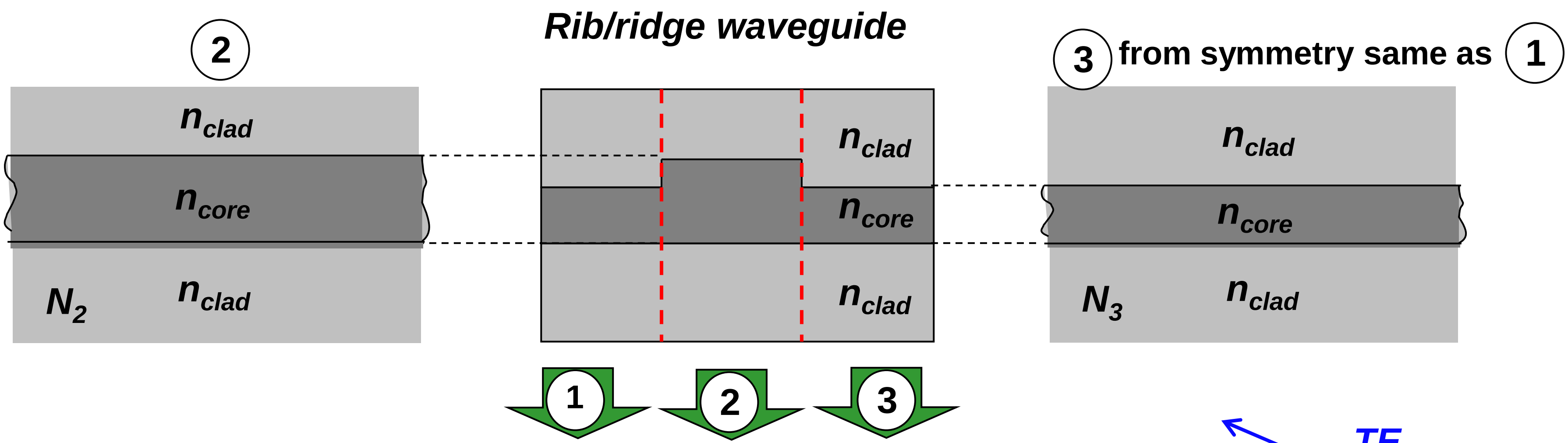
Any field can be expressed as their superposition

Any longitudinally uniform structure has its eigen modes – no matter what is the cross-sectional shape

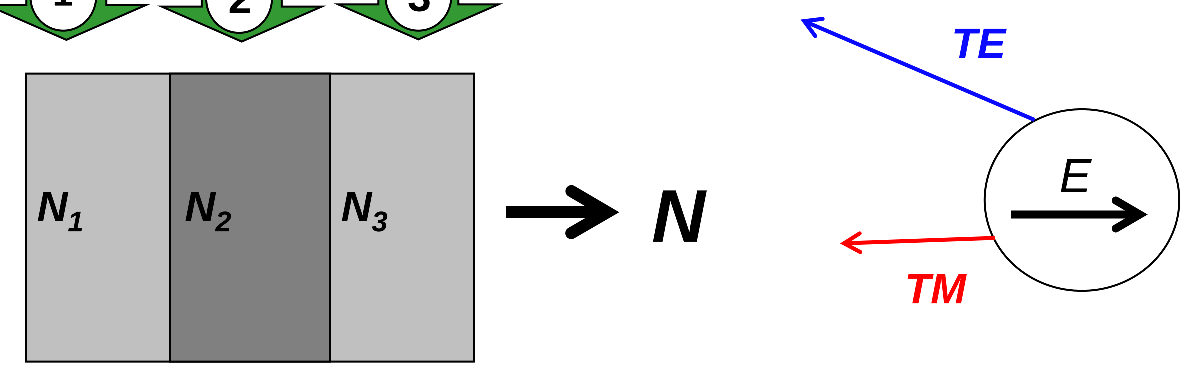
Coupling between modes is only possible when guided structure has a perturbation along mode propagation direction

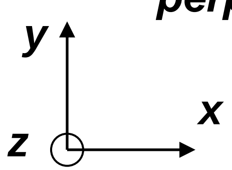
Effective index method (EIM)

Replace each of the rib three sections with a slab of the corresponding thickness.
Find effective indices for the slabs (1D problem, tabularized).
Replace the vertical (y) index profile in each section with the respective effective index.



Find effective index for the resulted "perpendicular" slab:

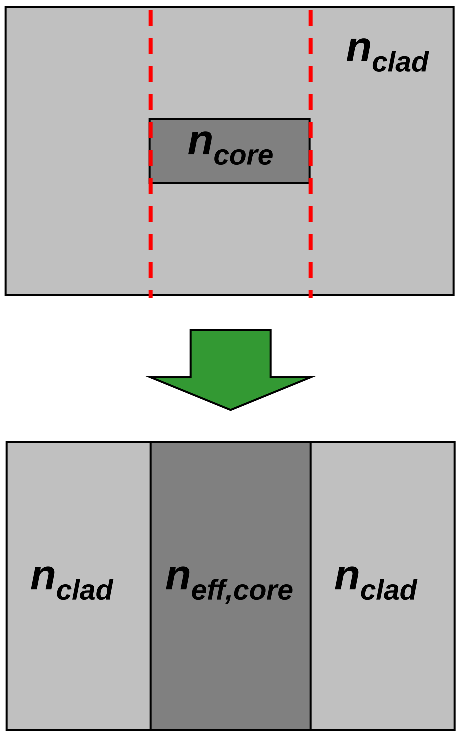




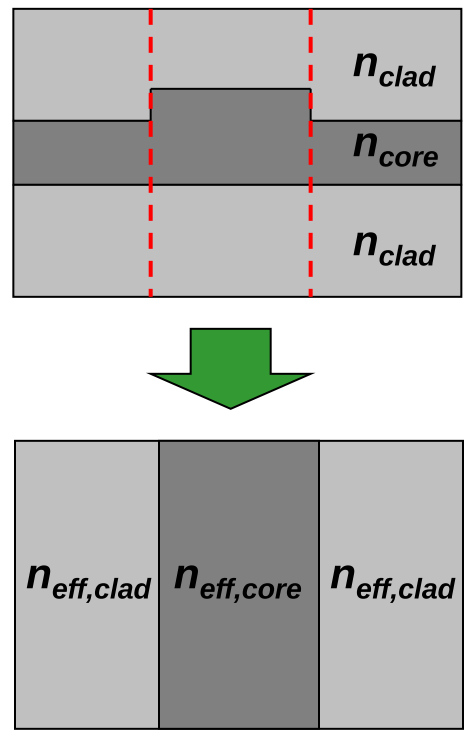
Note: EIM should be applied to each waveguide mode separately!
Change of polarization using the "perpendicular" reprezentation.

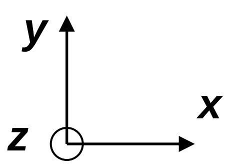
Effective index method

Channel waveguide



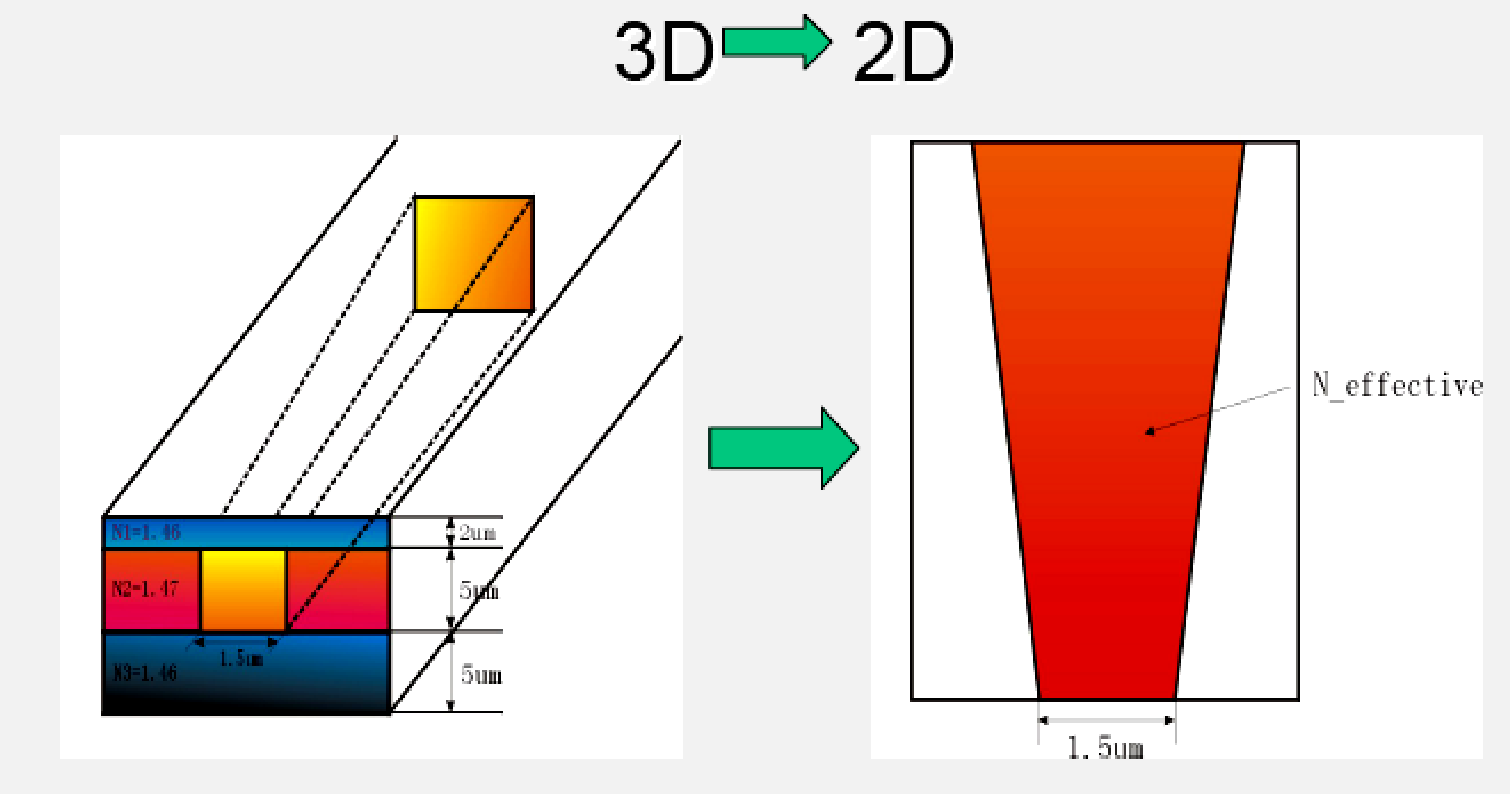
Rib/ridge waveguide





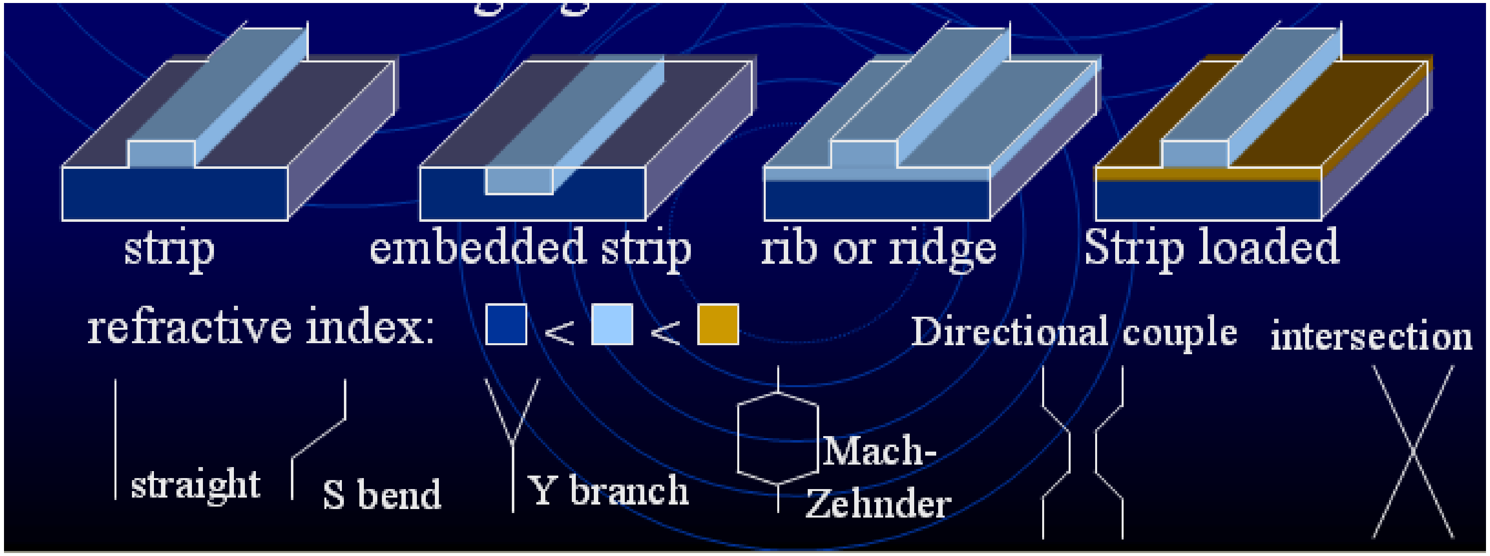
EIM mode solver - ridge waveguide:
http://wwwhome.math.utwente.nl/~hammerm/eims.html
http://wwwhome.math.utwente.nl/~hammer/eimsinout.html

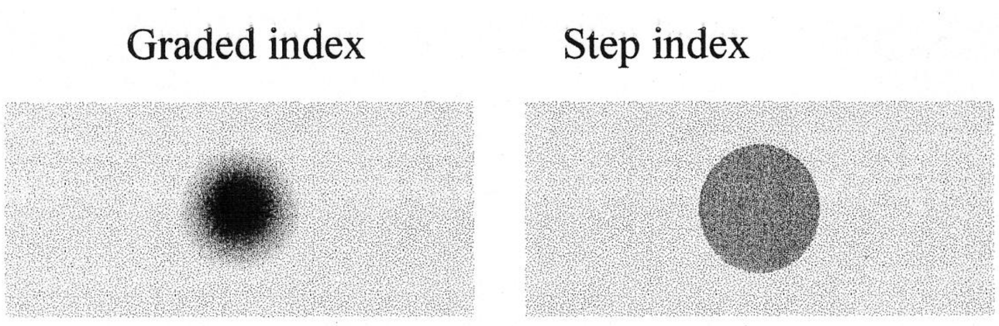
Effective Index Method



Reduces 3D problem to 2D one → Greatly improves computation efficiency

Geometries of channel waveguides





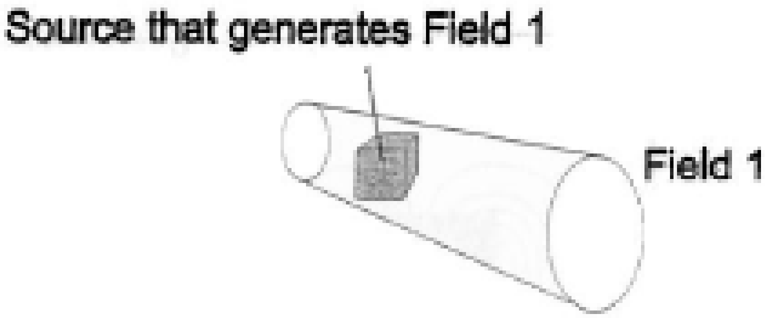
Coupled Mode Theory (CMT)

CTM is exact!

No para-axial approximation is needed to derive coupled wave equations

How we then approximate those equations depends on the physical problem

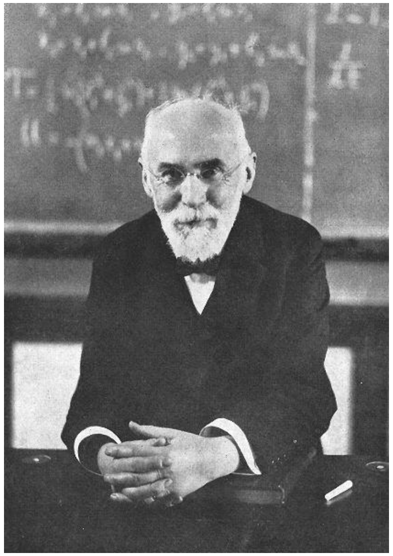
Lorentz Reciprocity Theorem



From Maxwell's equations, one can show that

$$\boxed{\nabla\cdot(\mathbf{E}_1\times\mathbf{H}_2^*+\mathbf{E}_2^*\times\mathbf{H}_1) = -j\omega\mathbf{P}_1\cdot\mathbf{E}_2^*}$$

$\mathbf{P}_1$ is the polarisation of the source that generates $\mathbf{E}_1$ and $\mathbf{E}_2$ is an arbitrary field.



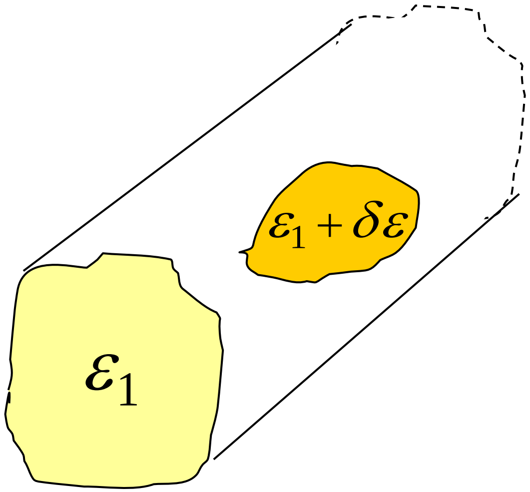
Hendrik Antoon Lorentz (1853 – 1928)

By integrating this relation, one can show that

$$\boxed{\iint dxdy\,\frac{\partial}{\partial z}(\mathbf{E}_1\times\mathbf{H}_2^*+\mathbf{E}_2^*\times\mathbf{H}_1)_z = -j\omega\iint dxdy\,\mathbf{P}_1\cdot\mathbf{E}_2^*}$$

Basis for Coupled Mode Theory

Polarization induced by perturbation of refractive index



$$\nabla \times H = j\omega \underbrace{\varepsilon}_{\varepsilon_1 + \delta\varepsilon} E$$

$$\boxed{\delta\varepsilon = \varepsilon_0 \delta n^2}$$

$$\nabla \times H = j\omega\varepsilon_1 E + j\omega \boxed{\delta\varepsilon\, E}$$

Source term

Induced polarization: $\boxed{P = \varepsilon_0 \delta n^2 E}$

***Perturbation polarization* can, for example, be induced by:**

- ***Waveguide defects***
- ***Introduced deformations, e.g. periodic ones***
- ***External electric DC field, e.g. by electro-optic effect:*** $\delta n \propto rE$
- ***Strong electromagnetic field, e.g. by optical Kerr effect:*** $\delta n \propto n_2 |E|^2$

Coupled mode equations (exact)

From the reciprocity theorem one can derive coupled equations for evolution of modal amplitudes due to refractive index perturbation

One assumes E_1 be a superposition of all modes and E_2 be a mode "μ" of unperturbed waveguide

Substituting E_1 and E_2 to the integral form of reciprocity theorem and making use of mode orthogonality yields:

Forward propagating mode

$$\boxed{\frac{d}{dz}A_\mu^+(z) = -\frac{j\omega}{4}\exp(j\beta_\mu z)\iint dxdy\; P\,E_\mu^*}$$

Backward Propagating mode

$$\boxed{\frac{d}{dz}A_\mu^-(z) = \frac{j\omega}{4}\exp(-j\beta_\mu z)\iint dxdy\; P\,E_{-\mu}^*}$$

$$P = \varepsilon_0 \delta n^2 \sum_\nu \boxed{A_\nu^+(z)E_\nu \exp(-j\beta_\nu z)} + \boxed{A_\nu^-(z)E_{-\nu}\exp(j\beta_\nu z)}$$

$\boxed{A_\nu^+(z)}$, $\boxed{A_\nu^-(z)}$ **Modal amplitudes slowly varying due to coupling**

Approximation - phase matched mode coupling

$$P = \varepsilon_0 \delta n^2 \sum_\nu A_\nu^+(z) E_\nu \exp(-j\beta_\nu z) + A_\nu^-(z) E_{-\nu} \exp(j\beta_\nu z)$$

$$\frac{d}{dz} A_\mu(z) = -\frac{j\omega\varepsilon_0}{4} \frac{\beta_\mu}{|\beta_\mu|} \exp(j\beta_\mu z) \iint \delta n^2 E_\mu^* \sum_\nu A_\nu(z) \exp(-j\beta_\nu z) E_\nu$$

Note phase factors !

Non negligible contribution from the RHS only if oscillations of the total phase factor ≈ 0

➡ ***Only those modes can efficiently couple for which the phase mismatch is compensated by the perturbation***

From phase matching condition usually most of the modes can be eliminated

Coupling between two modes

Co-directional

$$\frac{dA_m}{dz} = -i\kappa\, A_l\, e^{i\Delta\beta\, z}$$

$$\frac{dA_l}{dz} = -i\kappa^* A_m e^{-i\Delta\beta\, z}$$

Contra-directional

$$\frac{dA_m}{dz} = -i\kappa\, B_l\, e^{i\Delta\beta\, z}$$

$$\frac{dB_l}{dz} = i\kappa^* A_m e^{-i\Delta\beta\, z}$$

Coupling coefficient: $\kappa = \frac{\omega\varepsilon_0}{4}\iint E_m^* \delta n^2 E_l\, dxdy$

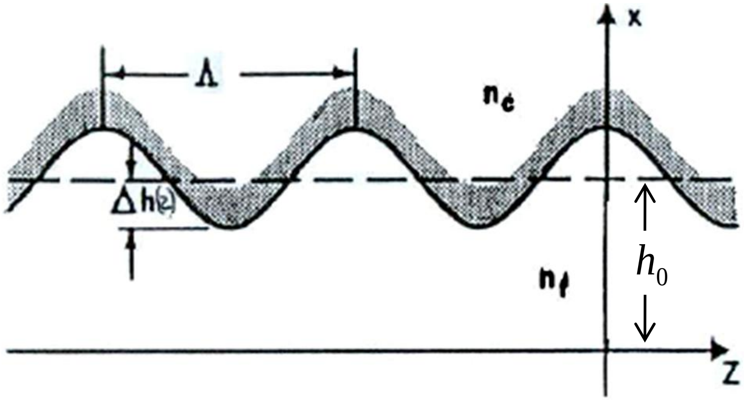
Phase mismatch:

$$\boxed{\Delta\beta = \beta_m - \beta_l}$$

$$\boxed{\Delta\beta = \beta_m + \beta_l}$$

Two phase mismatched modes can be coupled when z-variation of Δn^2 ***compensates the mismatch.***

Example: Mode coupling in periodic waveguide



Corrugated waveguide:

$$h(z) = h_0 + \Delta h \cos(\frac{2\pi}{\Lambda} z)$$

Regard this as distortion of waveguide of thickness h_0

$$\delta\varepsilon = \begin{cases} \varepsilon_0(n_f^2 - n_c^2) & for \quad h(z) > h_0 \\ -\varepsilon_0(n_f^2 - n_c^2) & for \quad h(z) < h_0 \end{cases}$$

Chose the modulation period so that it at some frequency matches the forward and backward modes of the same order (mode order skipped)

Periodic groves - Coupling coefficient

Evaluate the coupling coefficient : $\hat{\kappa}(z) = \frac{\omega}{4}\int E_m^* \delta\varepsilon(z) E_l \, dx$

Coupling between forward and backward modes of the same order: $E_m = E_l \equiv E$

$$\hat{\kappa} = \frac{\omega}{4}\int |E|^2 \delta\varepsilon \, dx \cong \frac{\omega}{4}|E(x=h_0)|^2 \delta\varepsilon \cdot 2 \int_0^{\Delta h \cos\frac{2\pi}{\Lambda}z} dx = \frac{\omega}{2}|E_h|^2 \delta\varepsilon \, \Delta h \cos\frac{2\pi}{\Lambda} z =$$

$$= \frac{\omega}{4}|E_h|^2 \delta\varepsilon \, (e^{i\frac{2\pi}{\Lambda}z} + e^{-i\frac{2\pi}{\Lambda}z}) = \kappa(e^{i\frac{2\pi}{\Lambda}z} + e^{-i\frac{2\pi}{\Lambda}z})$$

$$\kappa \equiv \frac{\omega}{4}|E_h|^2 \delta\varepsilon = \frac{\omega\varepsilon_0}{4}|E_h|^2 \delta n^2 = \frac{\omega\varepsilon_0}{4}|E_h|^2 2n\delta n = \frac{\omega\varepsilon_0 n}{2}|E_h|^2 \delta n$$

Forward to backward wave coupling

Chose the modulation period so that it at some frequency matches the forward and backward mode of the same order:

$$\frac{dA^+}{dz} = -i\kappa\, A^- \left[\cancel{e\left(i\frac{2\pi}{\lambda} z \right)} + e\left(-i\frac{2\pi}{\lambda} z \right) \right] e^{i(\beta_- + \beta_+)z}$$

$$\frac{dA^-}{dz} = i\kappa\, A^+ \left[e\left(i\frac{2\pi}{\Lambda} z \right) + \cancel{e\left(-i\frac{2\pi}{\Lambda} z \right)} \right] e^{-i(\beta_- + \beta_+)z}$$

Neglect the non phase-matched terms

Denote the phase mismatch by: $\Delta\beta \equiv \beta_- + \beta_+ - \dfrac{2\pi}{\Lambda}$

$$\beta_- = \beta_+ \quad \Rightarrow \quad \Delta\beta = 2\beta - \frac{2\pi}{\Lambda}$$

$$\Delta\beta = 0 \quad \Rightarrow \quad \boxed{2\beta = \frac{2\pi}{\Lambda} \quad \Rightarrow \quad \Lambda = \frac{\lambda}{2n_{eff}}}$$

Bragg condition

Solution for mode coupling

$$\boxed{\begin{aligned} \frac{dA^+}{dz} &= -i\kappa\, A^- e^{i2\Delta\beta z} \\ \frac{dA^-}{dz} &= i\kappa\, A^+ e^{-i2\Delta\beta z} \end{aligned}}$$

$$\oplus\ A^-(L) = 0$$

***L* –length of the corrugated section**

Solution:

$$A^+(z) = A^+(0)\ e^{i(\Delta\beta/2)z}\ \frac{s\cosh s(L-z) + i(\Delta\beta/2)\sinh s(L-z)}{s\cosh sL + i(\Delta\beta/2)\sinh sL}$$

$$A^-(z) = A^+(0)\ e^{-i(\Delta\beta/2)z}\ \frac{-i\kappa^*\sinh s(L-z)}{s\cosh sL + i(\Delta\beta/2)\sinh sL}$$

$$S = \sqrt{|\kappa|^2 - \left(\frac{\Delta\beta}{2}\right)^2} = \sqrt{|\kappa|^2 - [\beta(\omega) - \beta]^2}$$

Reflectance:

$$\boxed{R(s) = \left|\frac{A^-(0)}{A^+(0)}\right|^2 = \frac{|\kappa|^2\sinh^2 sL}{s^2\cosh^2 sL + (\Delta\beta/2)^2\sinh^2 sL}}$$

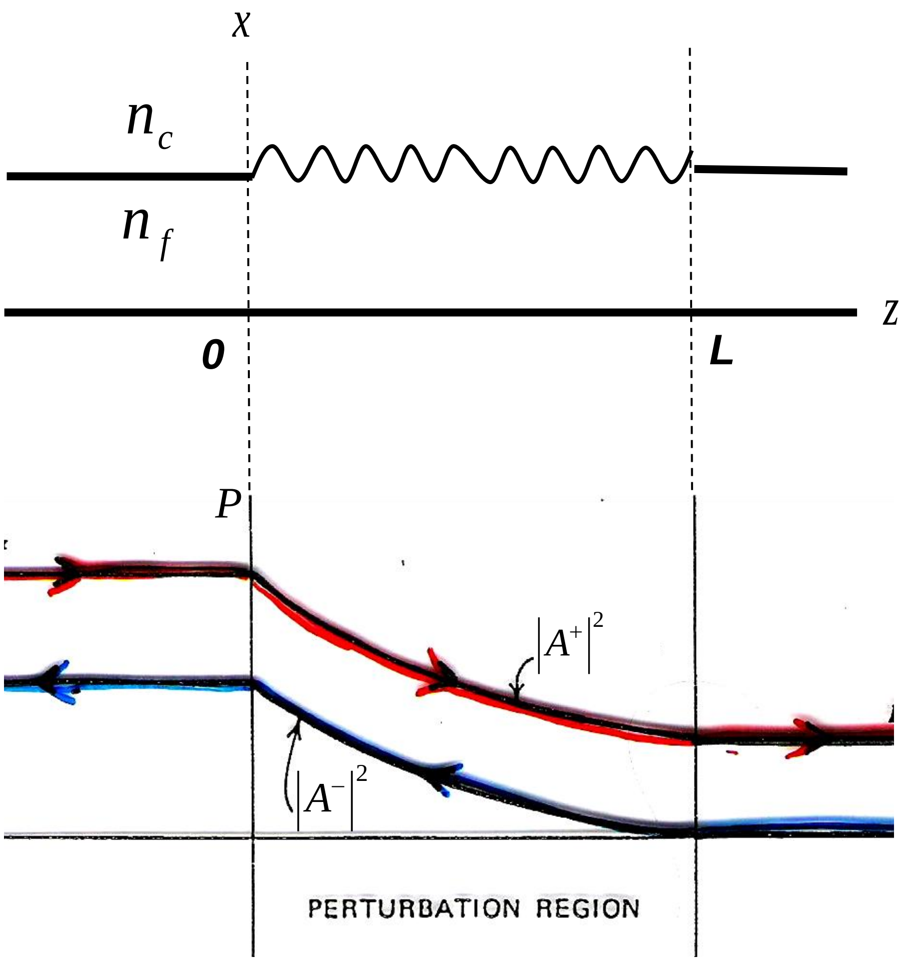
Solution at Bragg resonance

For $\Delta\beta$ ***= 0***

(Bragg condition):

$$A^{+}(z) = A^{+}(0)\frac{\cosh\left[|\kappa|(z-L)\right]}{\cosh\left[|\kappa|L\right]}$$

$$A^{-}(z) = A^{+}(0)\frac{-i\kappa\sinh\left[|\kappa|(z-L)\right]}{|\kappa|\cosh\left[|\kappa|L\right]}$$



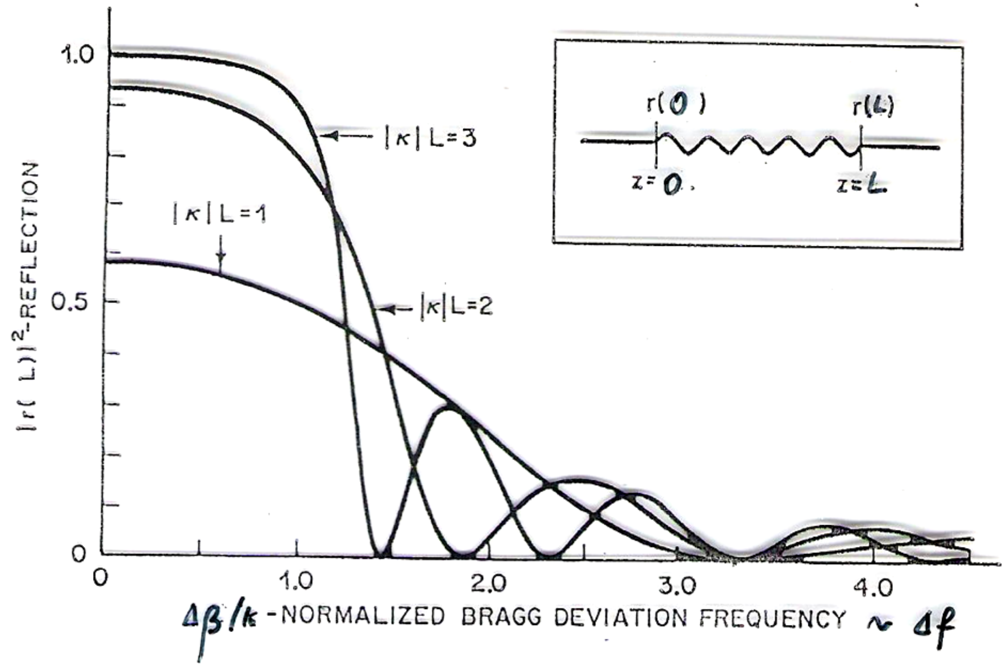
Reflectance:

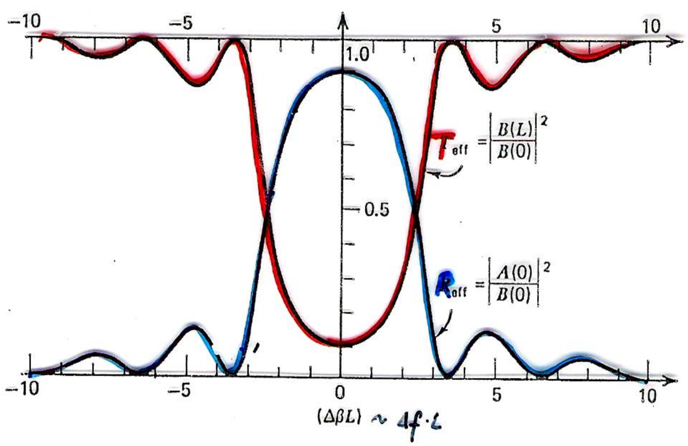
$$R_{\max} = \left|\frac{A^{-}(0)}{A^{+}(0)}\right|^{2} = \tanh^{2}|\kappa|L$$

Bandwidth for Bragg gratings

Note that length L is fixed here

The bandwidth for κL< 3 will shrink with decreasing L !

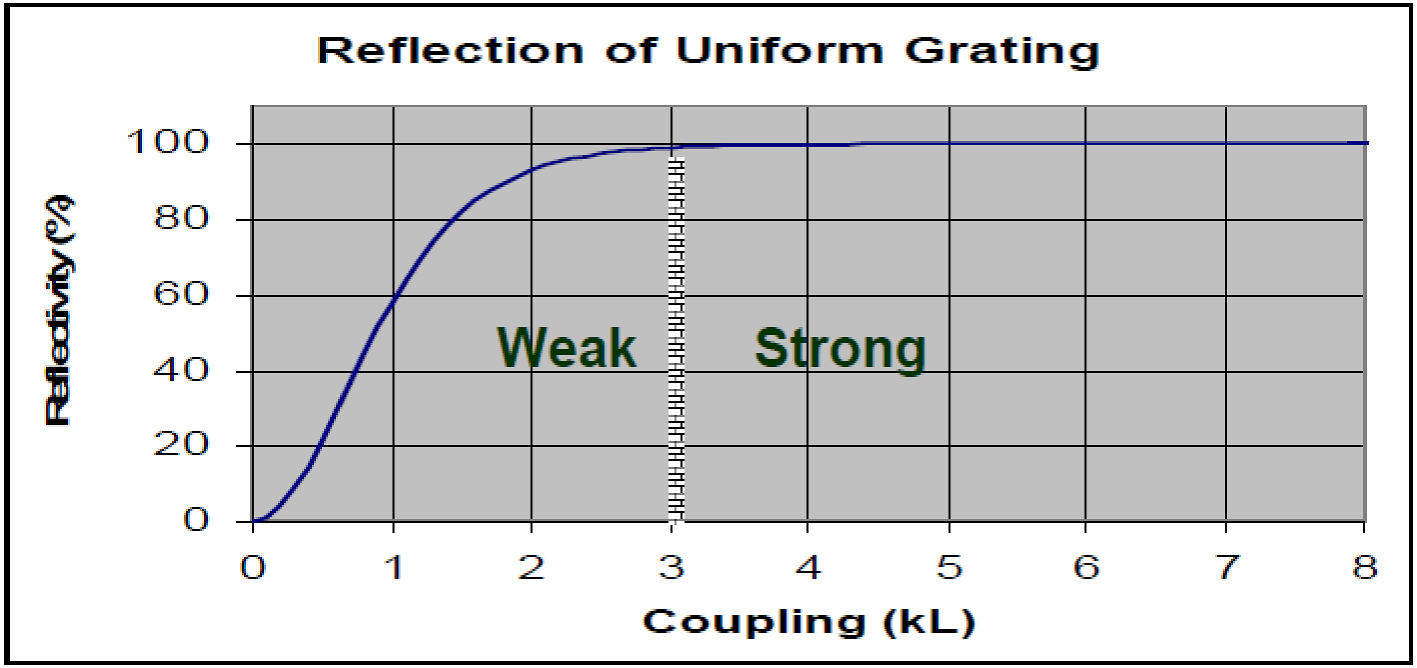




Grating reflectivity

$$R = \tanh^2(\kappa_{ac} L)$$

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

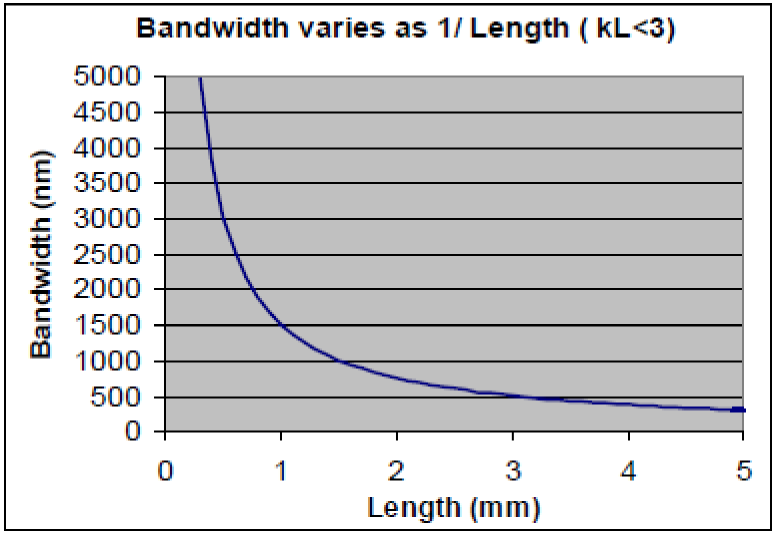


Bandwidth – weak grating

$$\Delta\lambda_{BW} \approx \frac{\lambda^2}{nL}$$

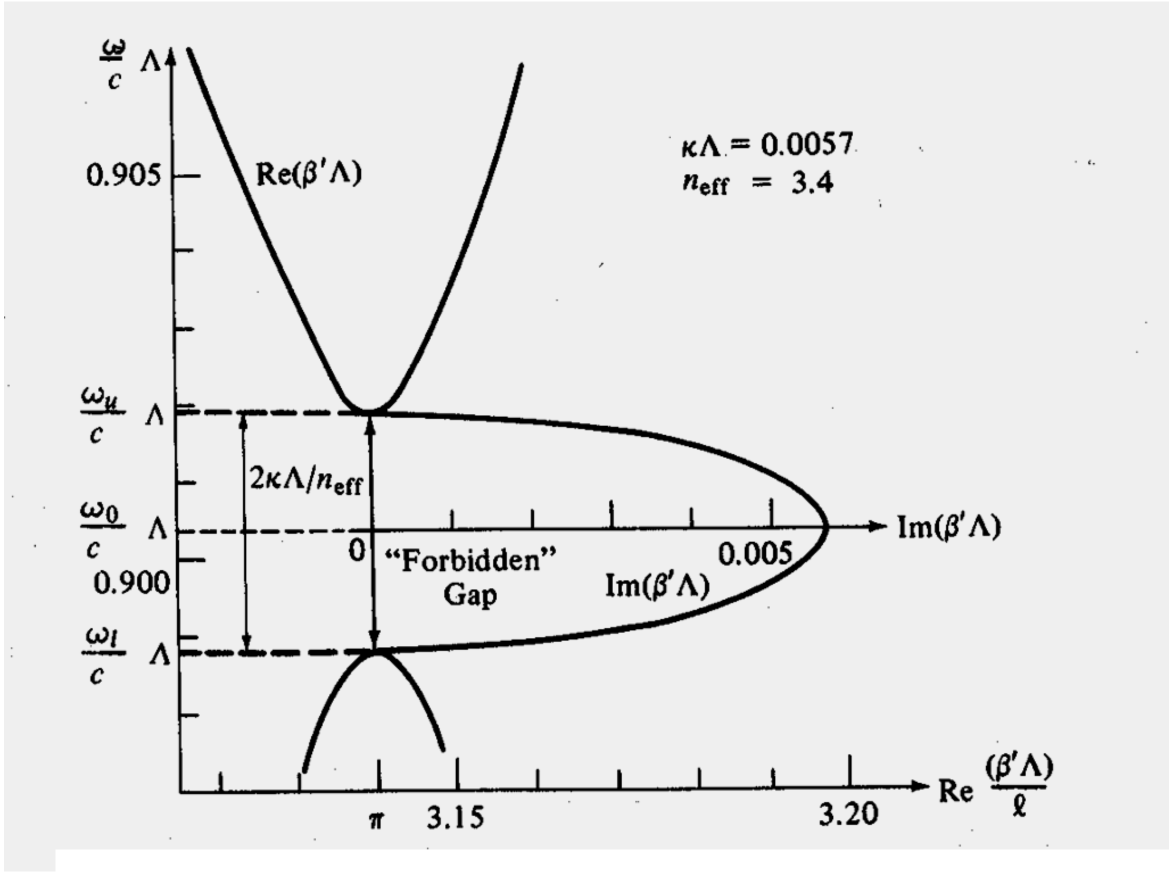
Bandwidth depends on L !

(decreases with L)



Weak, long gratings are used as wavelength filters

Photonic band gap due to periodicity



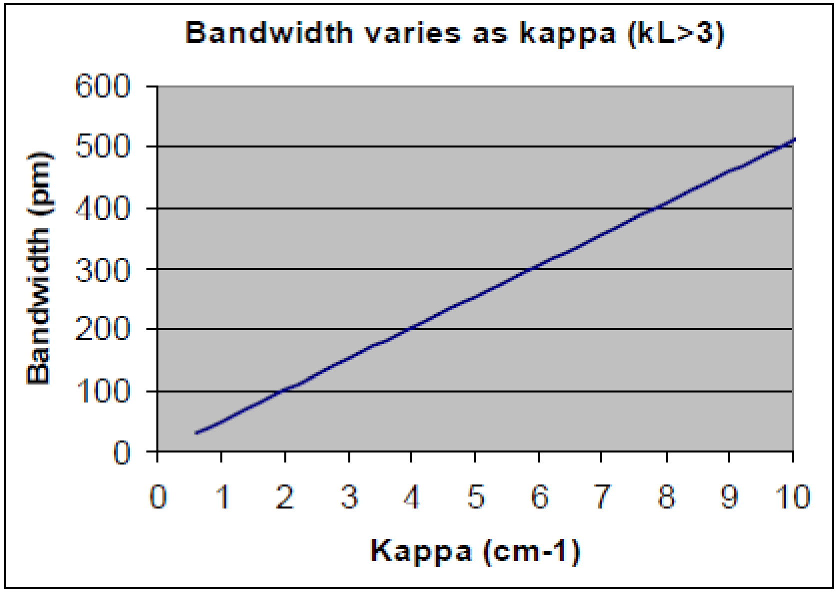
$$\left(\Delta\omega\right)_{gap} = \frac{2\kappa c}{n_{eff}}$$

Bandwidth – strong grating

$$\Delta\lambda_{BW} \approx \frac{\lambda^2}{\pi}\kappa_{ac}$$

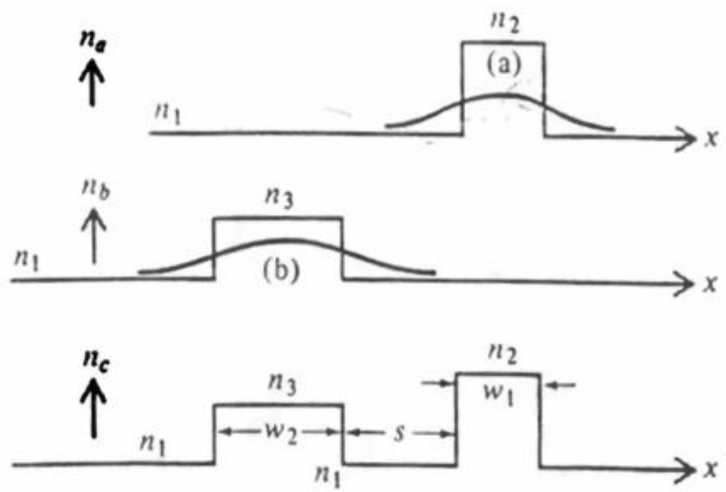
Bandwidth depends on κ_{ac} !

(increases with κ)

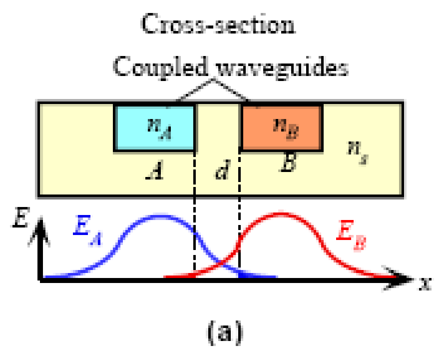


Strong gratings are used as broadband reflectors

Modes coupling in directional coupler



Yariv Chapt. 13



$$E_y = A(z)\mathcal{E}_y^{(a)}(x)e^{i(\omega t-\beta_a z)} + B(z)\mathcal{E}_y^{(b)}(x)e^{i(\omega t-\beta_b z)}$$

$$P_{\text{pert}} = e^{i\omega t}\varepsilon_0[\mathcal{E}_y^{(a)}A(z)(n_c^2(x) - n_a^2(x))e^{-i\beta_a z} + \mathcal{E}_y^{(b)}B(z)(n_c^2(x) - n_b^2(x))e^{-i\beta_b z}]$$

Arm a sees arm b as a perturbation

Arm b sees arm b as a perturbation

Coupled mode equations for directional coupler

$$\begin{cases} \dfrac{dA}{dz} = -i\kappa_{ab} B e^{-i(\beta_b-\beta_a)z} - iM_a A \\ \dfrac{dB}{dz} = -i\kappa_{ba} A e^{-i(\beta_a-\beta_b)z} - iM_b B \end{cases}$$

Coupling coefficient

$$\kappa_{ba}^{ab} = \frac{\omega\varepsilon_0}{4}\int_{-\infty}^{\infty}[n_c^2(x) - n_{(a,b)}^2(x)]\mathscr{E}_y^{(a)}\mathscr{E}_y^{(b)}\,dx$$

β correction

$$M_{(a,b)} = \frac{\omega\varepsilon_0}{4}\int_{-\infty}^{\infty}[n_c^2(x) - n_{(a,b)}^2(x)](\mathscr{E}_y^{(a,b)})^2\,dx$$

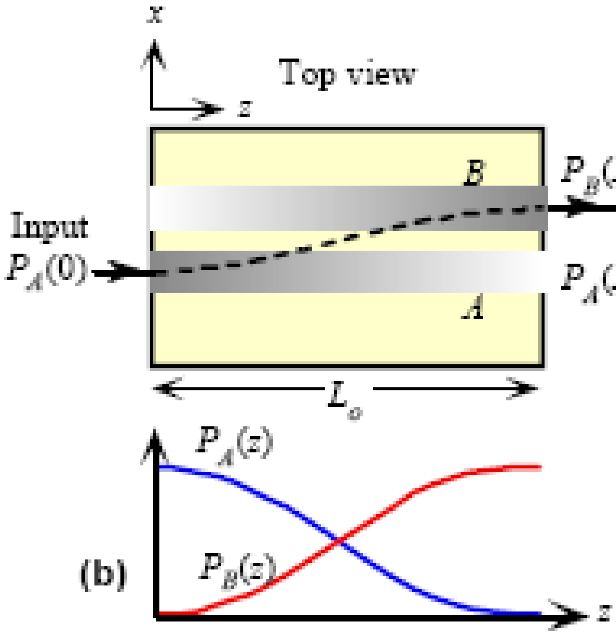
$$E_y = A(z)\mathscr{E}_y^{(a)}e^{i[\omega t-(\beta_a+M_a)z]} + B(z)\mathscr{E}_y^{(b)}e^{i[\omega t-(\beta_b+M_b)z]}$$

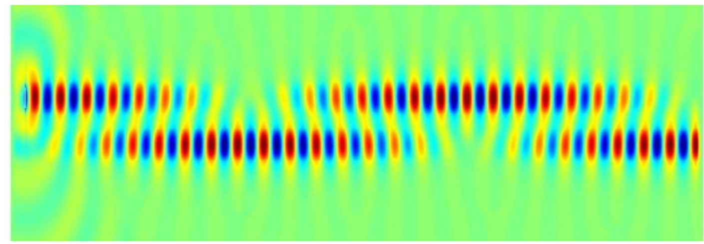
$$\begin{cases} \dfrac{dA}{dz} = -i\kappa_{ab} B e^{-i2\delta z} \\ \dfrac{dB}{dz} = -i\kappa_{ba} A e^{i2\delta z} \end{cases}$$

$$2\delta = (\beta_b + M_b) - (\beta_a + M_a)$$

$$\boxed{\begin{aligned} A(z) &= A_0 e^{i\delta z}\left(\cos sz - i\frac{\delta}{s}\sin sz\right) \\ B(z) &= -A_0 i e^{-i\delta z}\frac{\kappa}{s}\sin sz \\ \text{for: } & \kappa_{ab} = \kappa_{ba} \equiv \kappa \qquad s = \sqrt{\kappa^2 + \delta^2} \end{aligned}}$$

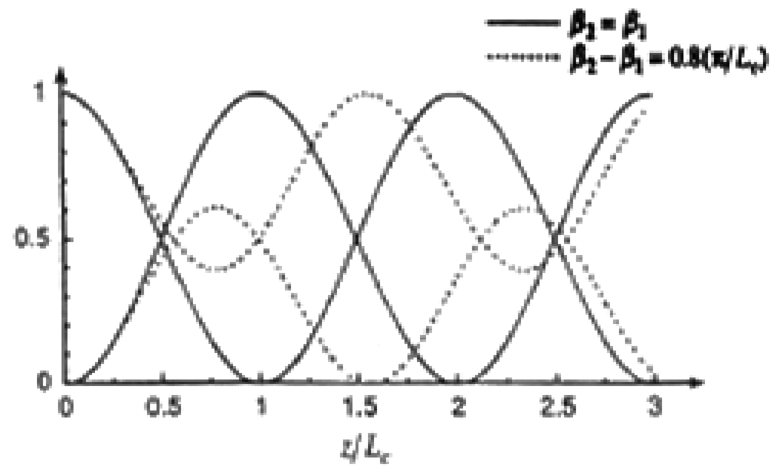
Directional coupler cont.





For $\delta \neq 0$, the maximum fraction of power that can be transferred

$$\frac{\kappa^2}{\kappa^2 + \delta^2}$$



$$P_A(z) = P_A(0) - P_B(z)$$

$$P_B(z) = P_A(0)\frac{\kappa^2}{\kappa^2+\delta^2}\sin^2\left(\sqrt{\kappa^2+\delta^2}\; z\right); \qquad 2\delta \cong \beta_A - \beta_B$$

Complete power transfer (δ=0): $\boxed{L_c = \pi/2\kappa}$

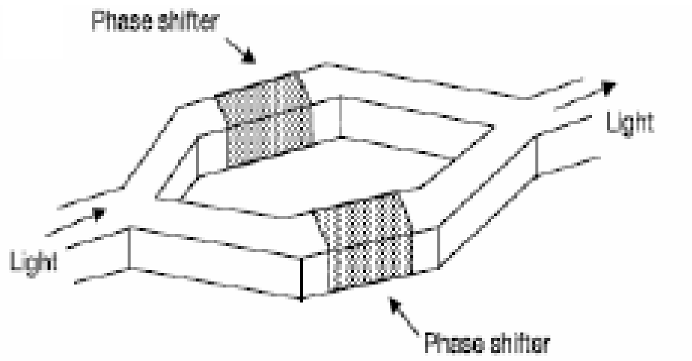
Coupling length

Coupling and Interference

Switches and multiplexers based on planar waveguides or fibers typically utilize:

Interference effects

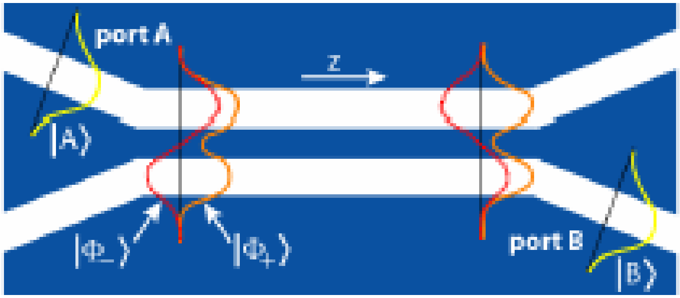
e.g. Fabry-Perot Interferometer
Mach-Zehnder interferometer



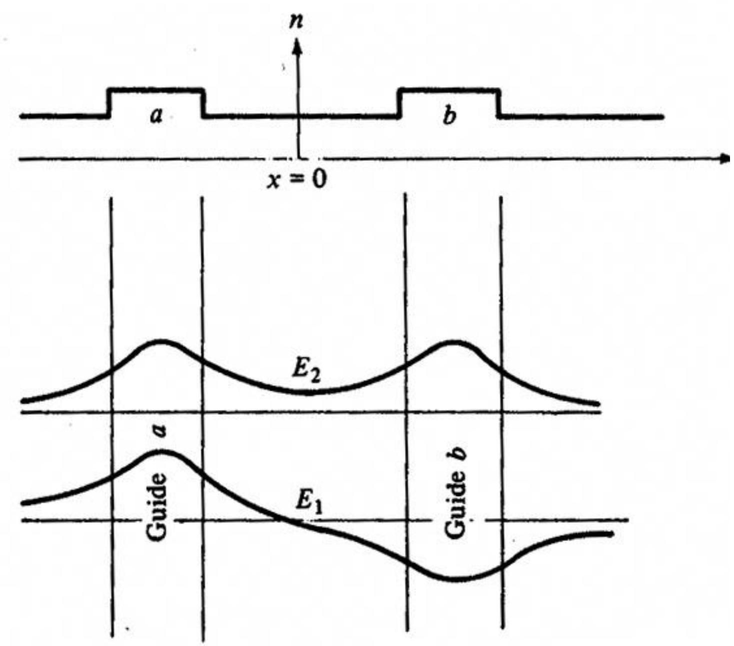
or

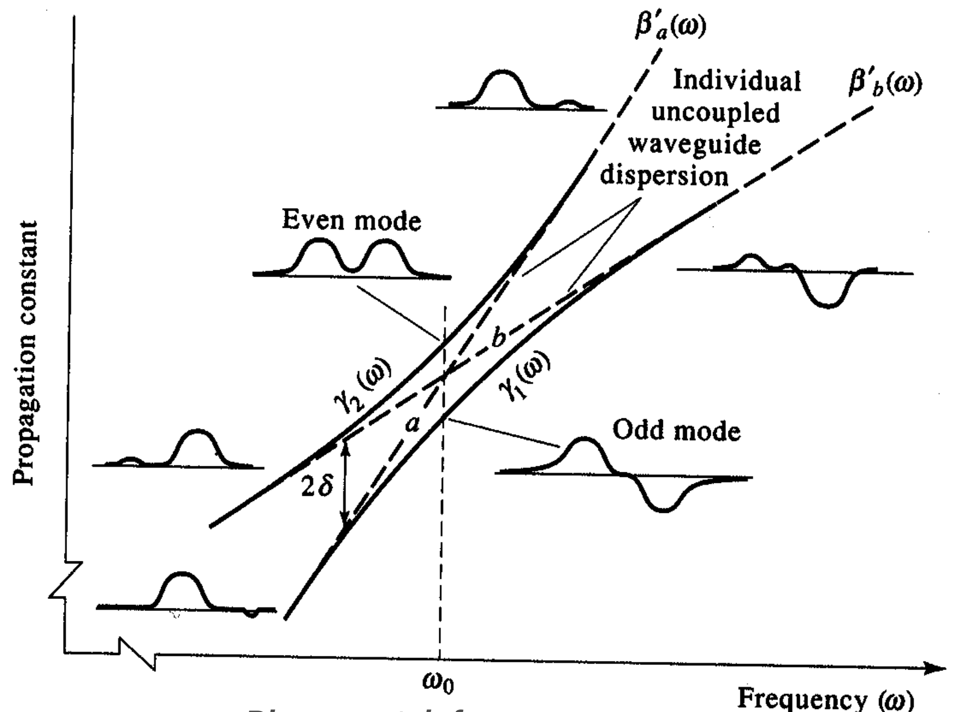
Distributed mode coupling

e.g. Bragg grating
Directional coupler ?



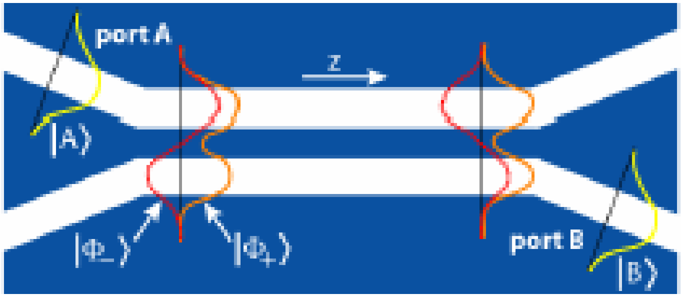
Directional coupler – Eigen (“super”) modes





Phase match frequency

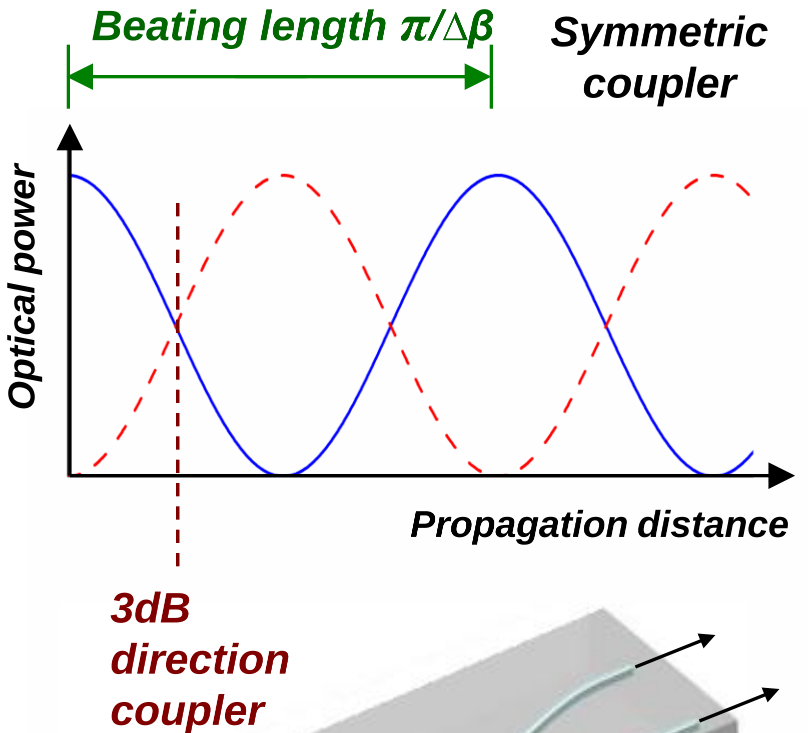
Interference of two modes

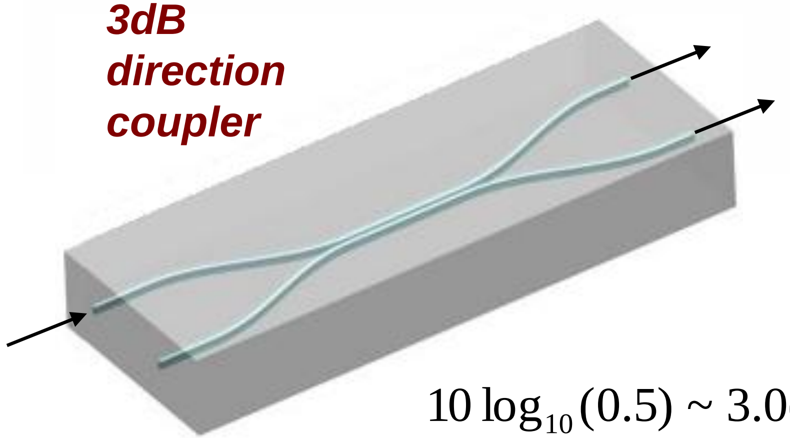


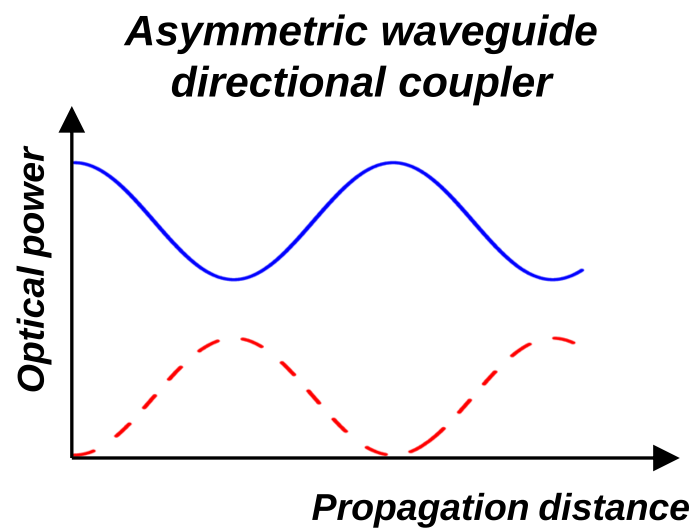
$$L_c = \frac{1}{2} L_B = \frac{\pi}{\Delta\beta} \equiv \frac{\pi}{|\beta_{odd} - \beta_{even}|}$$

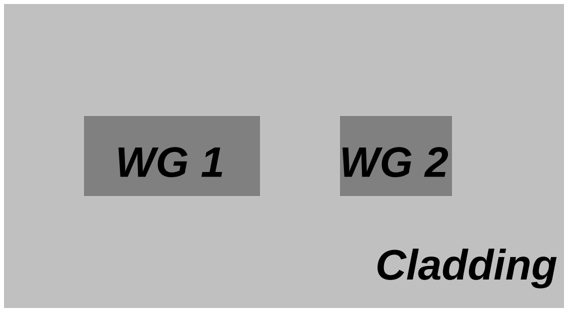
Coupling length = ½ Beat length for the supermodes

Directional coupler



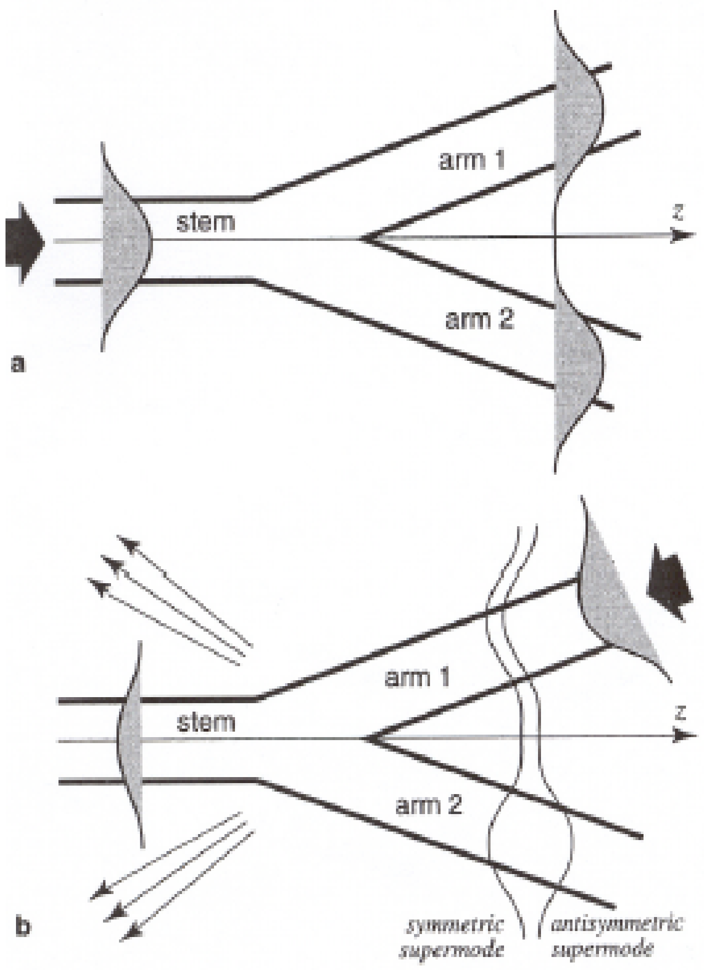






$$10\log_{10}(0.5) \sim 3.0(dB)$$

Y-junction



- Y-junctions can be used to split or combine signals
- They are found in a number of WDM components

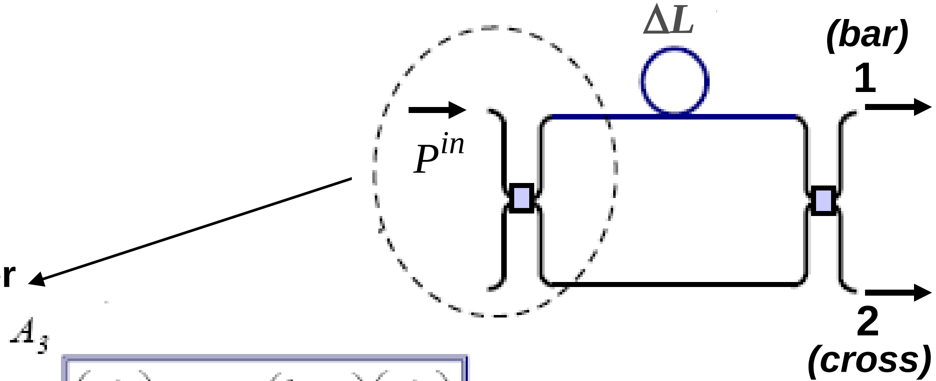
The functioning of Y-junctions can be discussed using the concept of supermodes:

splitting: the modal field distribution is adapting to the change in cross section (adiabatic transition). Power conservation occurs only if the change in cross-section is slow enough

backward direction: the mode can be decomposed in symmetric ands antisymmetric supermodes. The antisymmetric supermode is radiated at the junction. Half of the power is lost.

Mach Zehnder Interferometer (MZI) demultiplexer

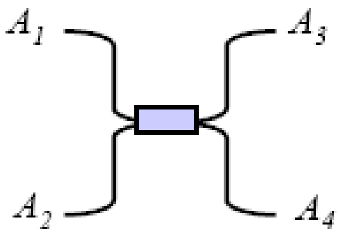
Consider both arms be identical waveguides but their length differs by ΔL



$$\frac{P_1^{out}}{P^{in}} = \sin^2\left(\frac{2\pi N(\Delta L/2)}{\lambda}\right)$$

$$\frac{P_2^{out}}{P^{in}} = \cos^2\left(\frac{2\pi N(\Delta L/2)}{\lambda}\right)$$

50:50 coupler



$$\begin{pmatrix} A_3 \\ A_4 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$$

There is a $\pi/2$ phase shift for the cross signal!

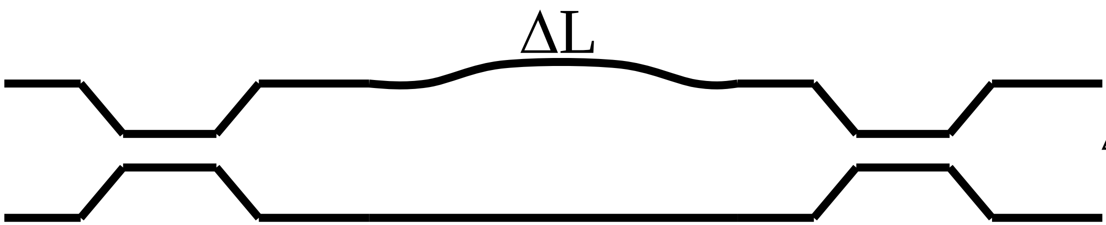
All the power at λ_1 *will exit from port 1 and all the power at* λ_2 *from port 2 if:*

$$\pi N\Delta L(1/\lambda_1) = \pi/2 \quad \text{and} \quad \pi N\Delta L(1/\lambda_2)] = \pi$$

Hence:

$$\boxed{\begin{aligned} \Delta L &= \{2n_{eff}[(1/\lambda_1) - (1/\lambda_2)]\}^{-1} \\ &= c/(2n_{eff}\Delta\nu) \end{aligned}}$$

MZI interleaver



$$\Delta\Phi = n_{eff} k_0 \Delta L$$

$$\begin{vmatrix} A_{1,out} \\ A_{2,out} \end{vmatrix} = \begin{vmatrix} \frac{1}{\sqrt{2}} & j\frac{1}{\sqrt{2}} \\ j\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{vmatrix} \begin{vmatrix} e^{j\Delta\Phi/2} & 0 \\ 0 & e^{-j\Delta\Phi/2} \end{vmatrix} \begin{vmatrix} \frac{1}{\sqrt{2}} & j\frac{1}{\sqrt{2}} \\ j\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{vmatrix} \begin{vmatrix} A_{1,in} \\ A_{2,in} \end{vmatrix} = \begin{vmatrix} \frac{1}{\sqrt{2}} & j\frac{1}{\sqrt{2}} \\ j\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{vmatrix} \begin{vmatrix} \frac{e^{j\Delta\Phi/2}}{\sqrt{2}} & j\frac{e^{j\Delta\Phi/2}}{\sqrt{2}} \\ j\frac{e^{-j\Delta\Phi/2}}{\sqrt{2}} & \frac{e^{-j\Delta\Phi/2}}{\sqrt{2}} \end{vmatrix} \begin{vmatrix} A_{1,in} \\ A_{2,in} \end{vmatrix} =$$

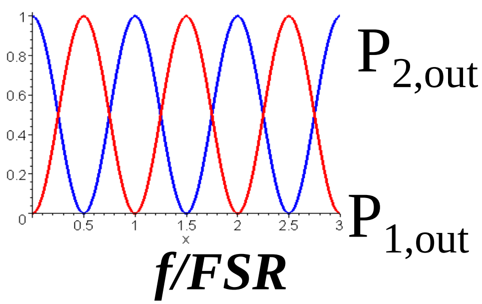
$$\begin{vmatrix} j\sin(\Delta\Phi/2) & j\cos(\Delta\Phi/2) \\ j\cos(\Delta\Phi/2) & -j\sin(\Delta\Phi/2) \end{vmatrix} \begin{vmatrix} A_{1,in} \\ A_{2,in} \end{vmatrix}$$

For: $A_{1,in}=A_0$ and $A_{2,in}=0$

$$FSR = c / n_{eff} \Delta L$$

$$P_{1,out} = P_0 \sin^2(\Delta\Phi/2) = P_0 \sin^2(\pi n_{eff} \Delta L f / c) = P_0 \sin^2(\pi f / FSR);$$

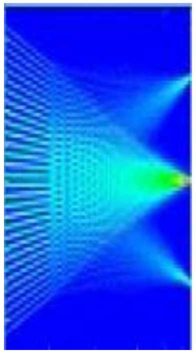
$$P_{2,out} = P_0 \cos^2(\Delta\Phi/2) = P_0 \cos^2(\pi n_{eff} \Delta L f / c) = P_0 \cos^2(\pi f / FSR);$$



Frequency silicer / DWDM interleavers - ***separates a series of optical channels so alternating wavelengths emerge out its two ports***

Arrayed Waveguide Grating de-multiplexer

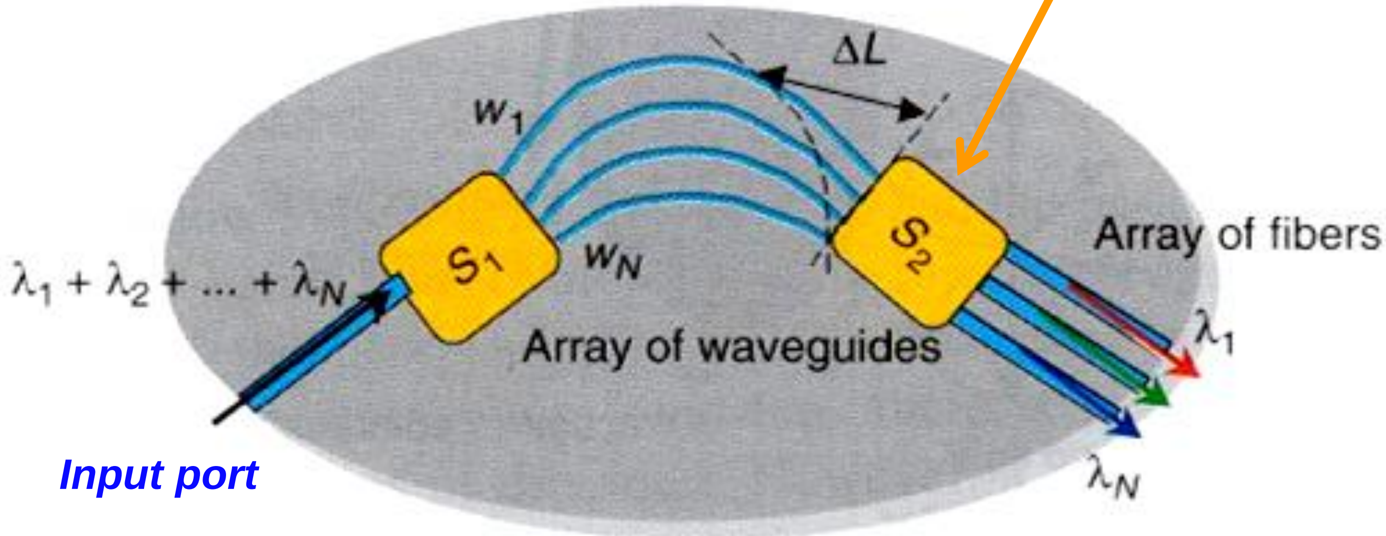
Interference ***separates wavelengths here***



λ_1

λ_2

λ_3



Input port

S_1 , S_2 - „star couplers“ or free space couplers

The coupling behavior of coupler S_2 depends on both λ_n and the location of the input port (which determines phase delay in S_1)

Picture of AWG from S.V. Kartalopoulos