WARSAW UNIVERSITY OF TECHNOLOGY
DEVELOPMENT PROGRAMME

IOP Institute of Physics

19th Ion Beam Analysis IBA

UNIVERSITY OF CAMBRIDGE

Wednesday 17th March, 2010: 11:00 -13:00

Imaging Systems Modelling

J M Blackledge

Stokes Professor
Dublin Institute of Technology
http://eleceng.dit.ie/blackledge

Distinguished Professor
Warsaw University of Technology

science foundation ireland
fondúireacht eolaíochta éireann

HUMAN CAPITAL
NATIONAL COHESION STRATEGY

EUROPEAN UNION
EUROPEAN
SOCIAL FUND

Lectures co-financed by the European Union in scope of the European Social Fund

What is the Problem?

- Fundamental signal/imaging equation is

$$s = \mathcal{L}f + n$$

s - signal/image
f - information
n - noise
$\mathcal{L}$ - linear operator

- ***Operator***, ***information*** and ***noise*** relate to ***physical effects:***

  - what are they?

  - how can we generate a physical model for them?

Applications

- ***Processing*** and ***analysis*** of all signal and images generated by the scattering of radiation from ***material inhomogeneities***

- Required to develop mathematical and computational models that map ***material inhomogeneities*** to the ***detected radiation***

- Area of study called ***Image Understanding***

Lecture will Focus on Electromagnetic Imaging Systems

$$
\begin{array}{ccccc}
 & & \text{Maxwell's equation} & & \\
 & & \downarrow & & \\
\text{Boundary Conditions} & \rightarrow & \text{Wave equation} & \leftarrow & \text{Green's function} \\
 & & \downarrow & & \\
\text{Geometry} & \rightarrow & \text{Integral equation} & \leftarrow & \text{Approximation} \\
 & & \downarrow & & \\
 & & \text{Imaging equation} & &
\end{array}
$$

Wavelength >> Scatterer (***weak scattering***)
Wavelength ~ Scatterer (***strong scattering***)
Wavelength << Scatterer (***geometric scattering***)

Principal Publication

http://eleceng.dit.ie/arg/downloads/PhDJMB2010.zip

JYVÄSKYLÄ STUDIES IN COMPUTING

108

Jonathan Blackledge

Electromagnetic Scattering Solutions for Digital Signal Processing

JYVÄSKYLÄN YLIOPISTO

UNIVERSITY OF JYVÄSKYLÄ

FACULTY OF INFORMATION TECHNOLOGY

Jonathan Blackledge (born 25th June 1959)

has been awarded the degree of

THE DEGREE OF DOCTOR OF PHILOSOPHY

as specified in the Finnish Government Decree No. 794/2004

Scientific postgraduate studies have been fulfilled in the field of Mathematical Information Technology.

The dissertation "Electromagnetic Scattering Solutions for Digital Signal Processing" has been defended at its public examination on January 28th, 2010. Professor Michael Rycroft (International Space University, France) and professor Timo Hämäläinen (University of Jyväskylä) as the chairman.

The Faculty Council of the Faculty of Information Technology has given the thesis the grade "approved with honours" on February 24th, 2010.

Jyväskylä, February 24th, 2010

P. Neittaanmäki

Pekka Neittaanmäki
Dean

Eija Ihanainen
Head of Academic Affairs

JYVÄSKYLÄN YLIOPISTO AMICA VERITAS

An English version of the Finnish certificate of the degree of Ph.D. FT 1/2010 (Filosofian tohtori)

Contents of Presentation I

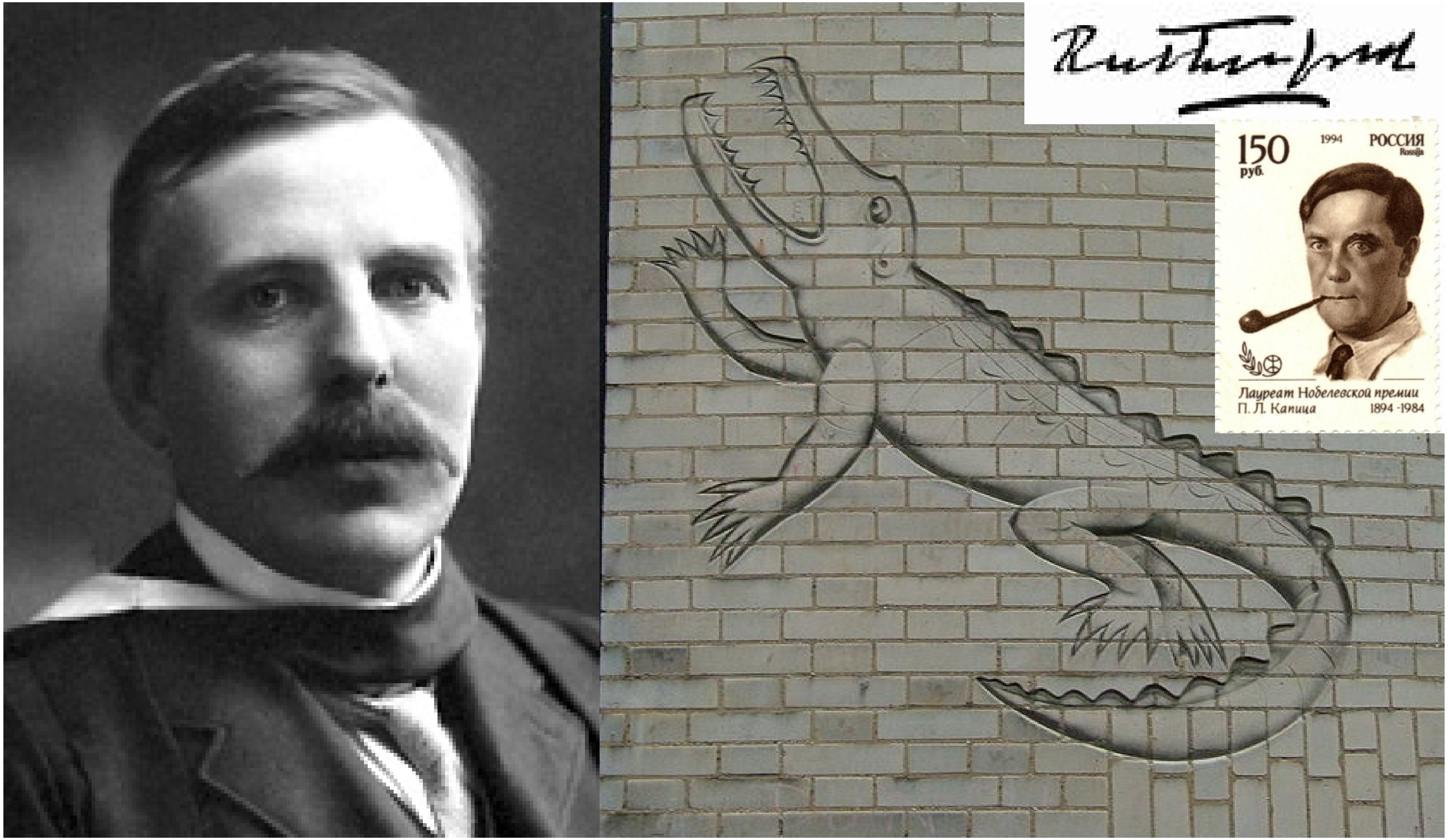
Part I:

- Rutherford Scattering
- Quantum Scattering Theory
- The Green's function
- The Lippmann-Schwinger Equation
- The Born Approximation
- Electromagnetic Scattering Theory
- Summary
- Q & A + Interval (10 Minutes)

Contents of Presentation II

- The Hubble Space Telescope and Einstein rings
- Low frequency scattering theory
- Why is an Einstein ring blue?
- Compatibility with General Relativity
- What is Gravity?
- The field equations of physics
- The Maxwell-Proca equations
- Summary
- Q & A

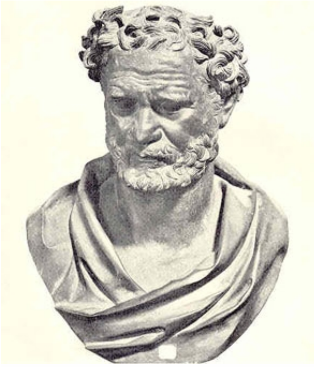
Rutherford Scattering



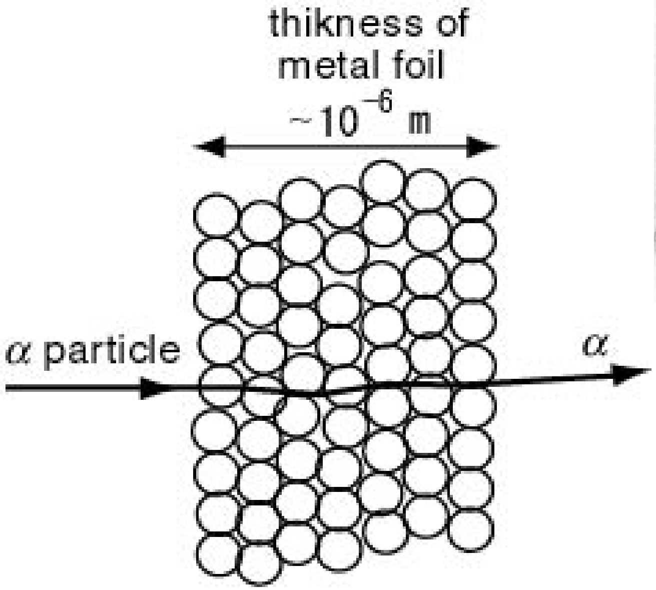
What did Rutherford do?

Initiated the study of particle physics:

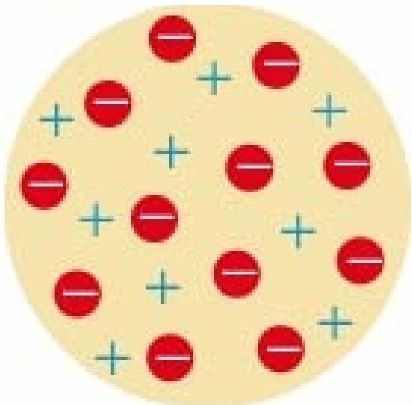
Rutherford 1909 - LHC (CERN) 2009



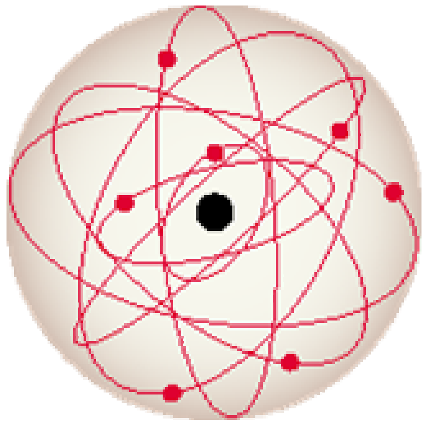
$$\text{Gaussian}\left(\frac{\theta}{2}\right)$$



$$\sin^{-4}\left(\frac{\theta}{2}\right)$$



ANIMATION



From England to Germany, Cambridge to Gottingen & From Particles to Waves

Why Gottingen and not Cambridge?

Max Born

Robert Oppenheimer

The Power of *Abstract Ideas*

Quantum Scattering Theory

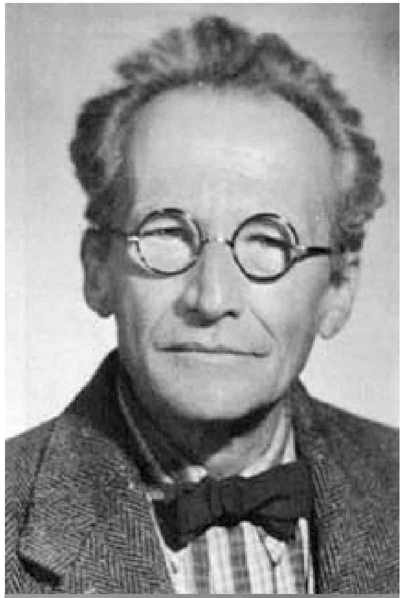
- *Schrodinger's equation*

  for a 3D scattering potential V is

$$(\nabla^2 + k^2)\psi(\mathbf{r},k) = -V(\mathbf{r})\psi(\mathbf{r},k)$$

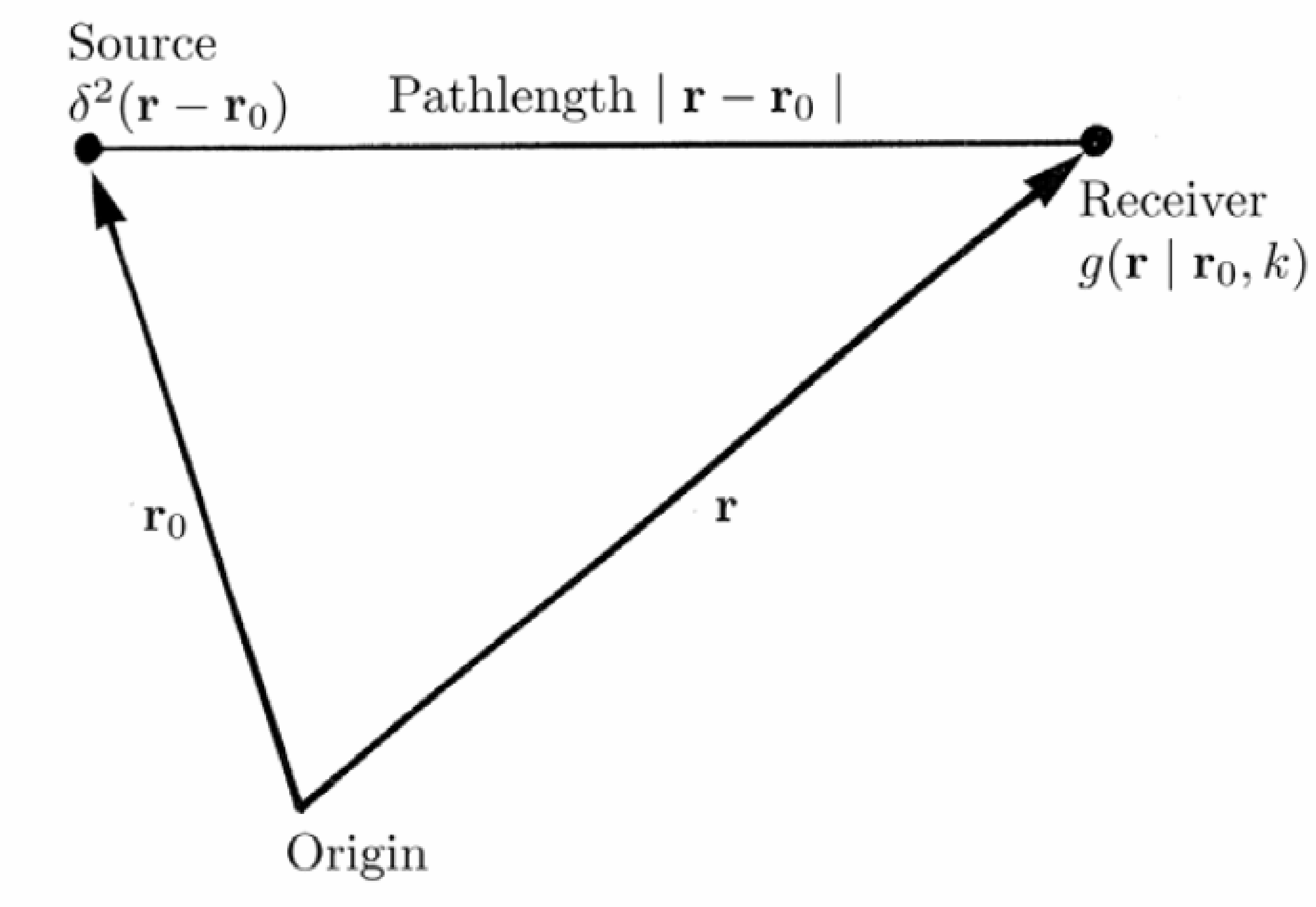
- Solved using the Green's function

$$g(r,k) = \frac{\exp(ikr)}{4\pi r}$$



What is a Green's Function? (Impulse Response Function)

$$(\nabla^2 + k^2) g(\mathbf{r} \mid \mathbf{r}_0, k) = -\delta^2(\mathbf{r} - \mathbf{r}_0)$$



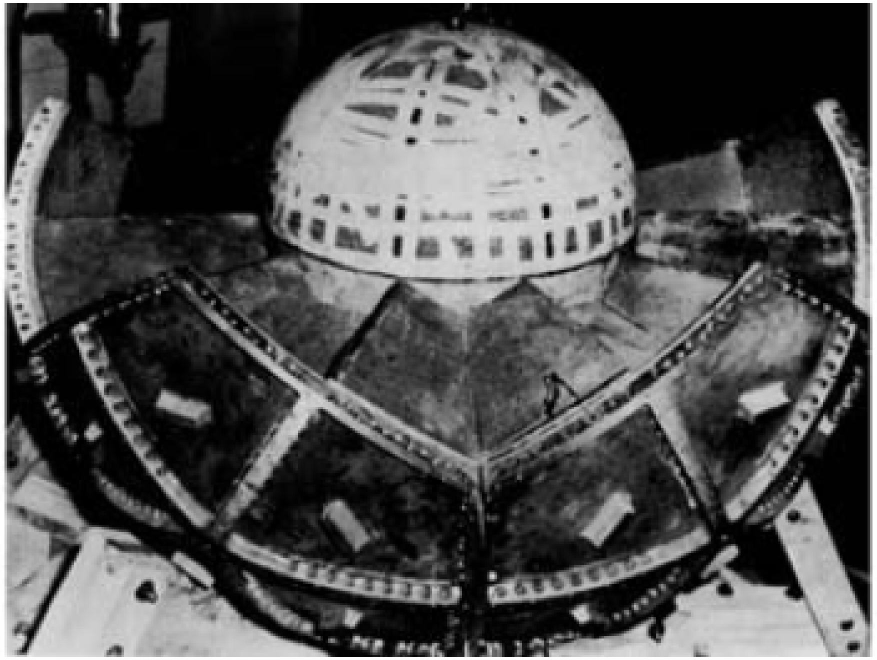
2D Example of a Green's Function

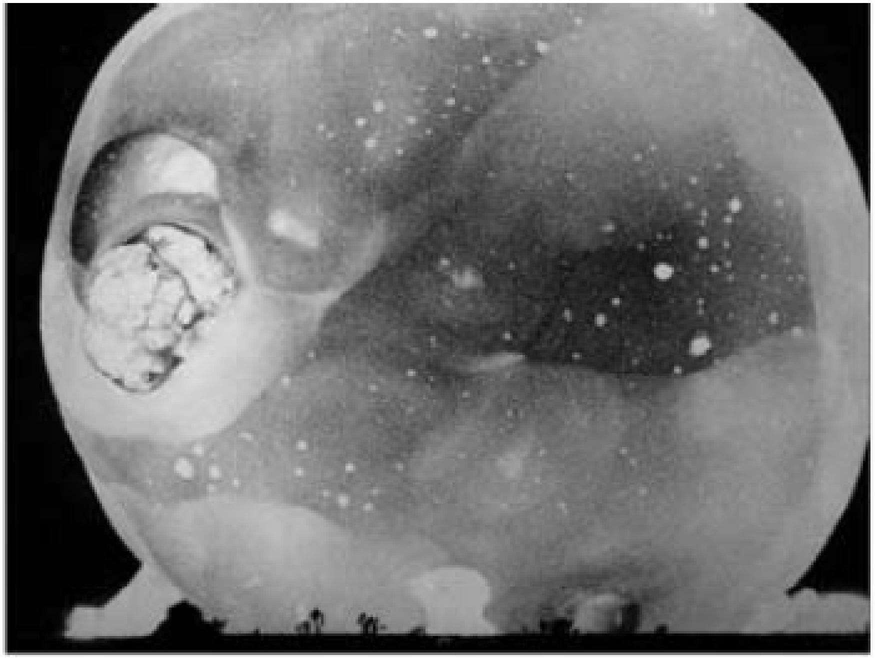
The 3D Green's Function

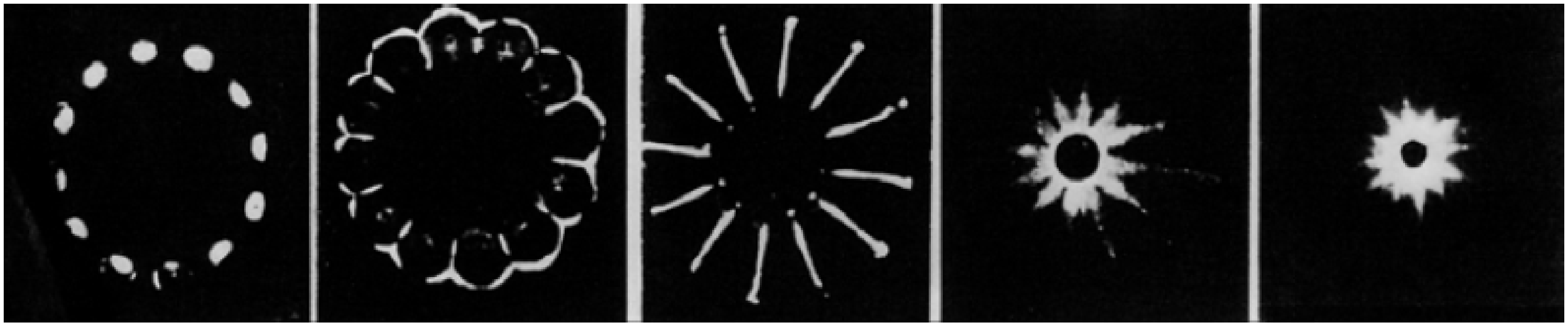
$$(\nabla^2 + k^2) g(\mathbf{r} \mid \mathbf{r}_0, k) = -\delta^3(\mathbf{r} - \mathbf{r}_0)$$

$$g(\mathbf{r} \mid \mathbf{r}_0, k) = \frac{1}{4\pi \mid \mathbf{r} - \mathbf{r}_0 \mid} \exp(ik \mid \mathbf{r} - \mathbf{r}_0 \mid)$$

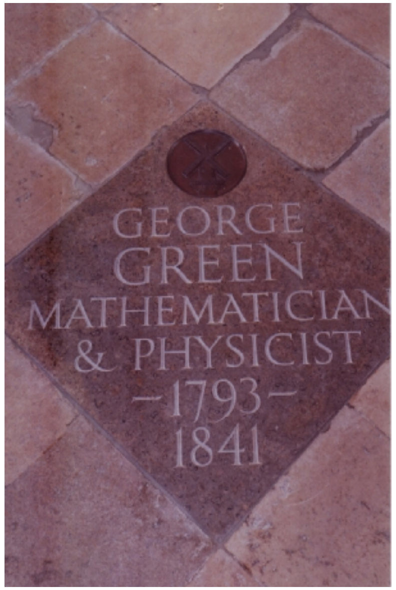
‘Outgoing’ and ‘Ingoing’ Green’s Functions







Who was George Green?



AN ESSAY

ON THE

APPLICATION

MATHEMATICAL ANALYSIS TO THE THEORIES OF ELECTRICITY AND MAGNETISM.

BY

GEORGE GREEN.

Nottingham:

PRINTED FOR THE AUTHOR, BY T. WHEELHOUSE.

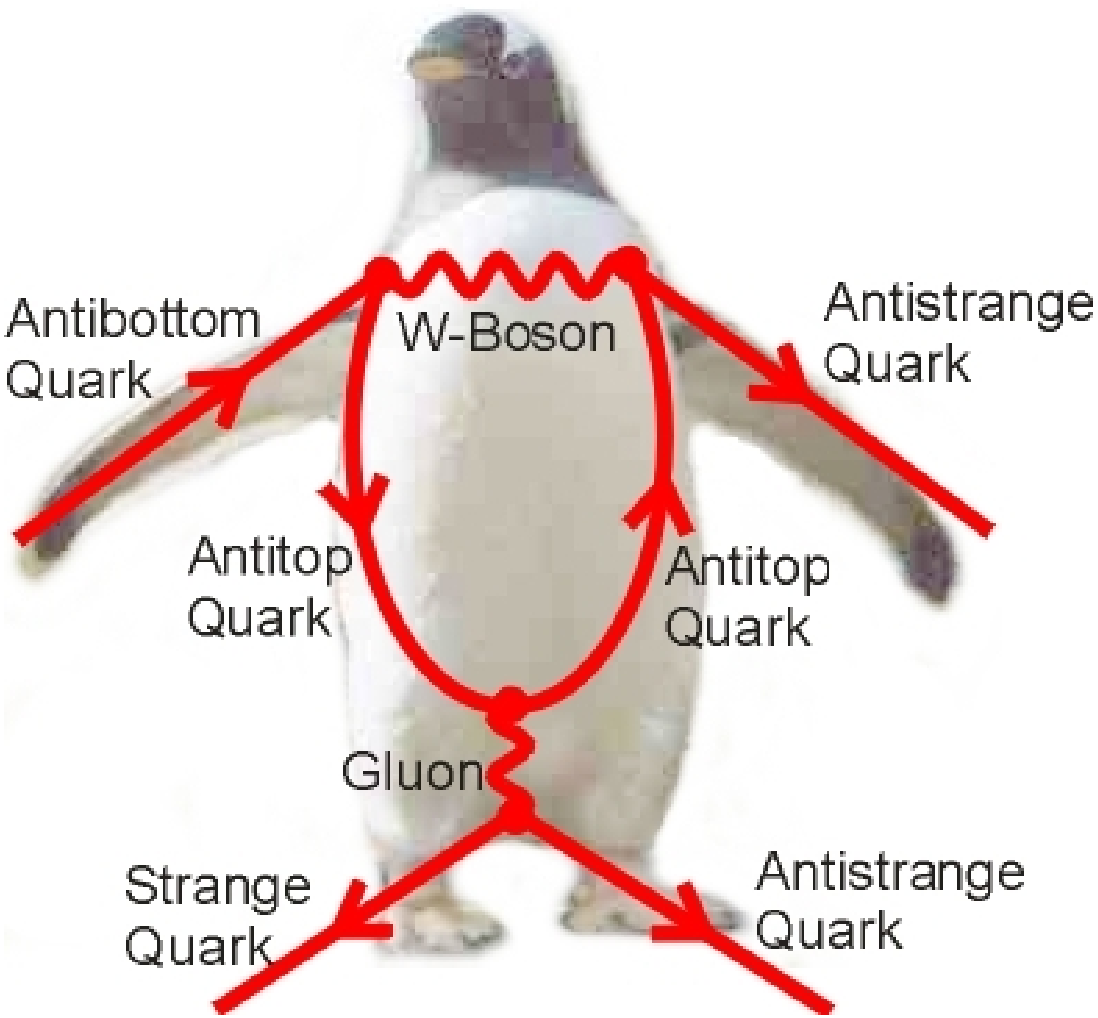
SOLD BY HAMILTON, ADAMS & Co. 33, PATERNOSTER ROW; LONGMAN & Co.; AND W. JOY, LONDON; J. DEIGHTON, CAMBRIDGE;

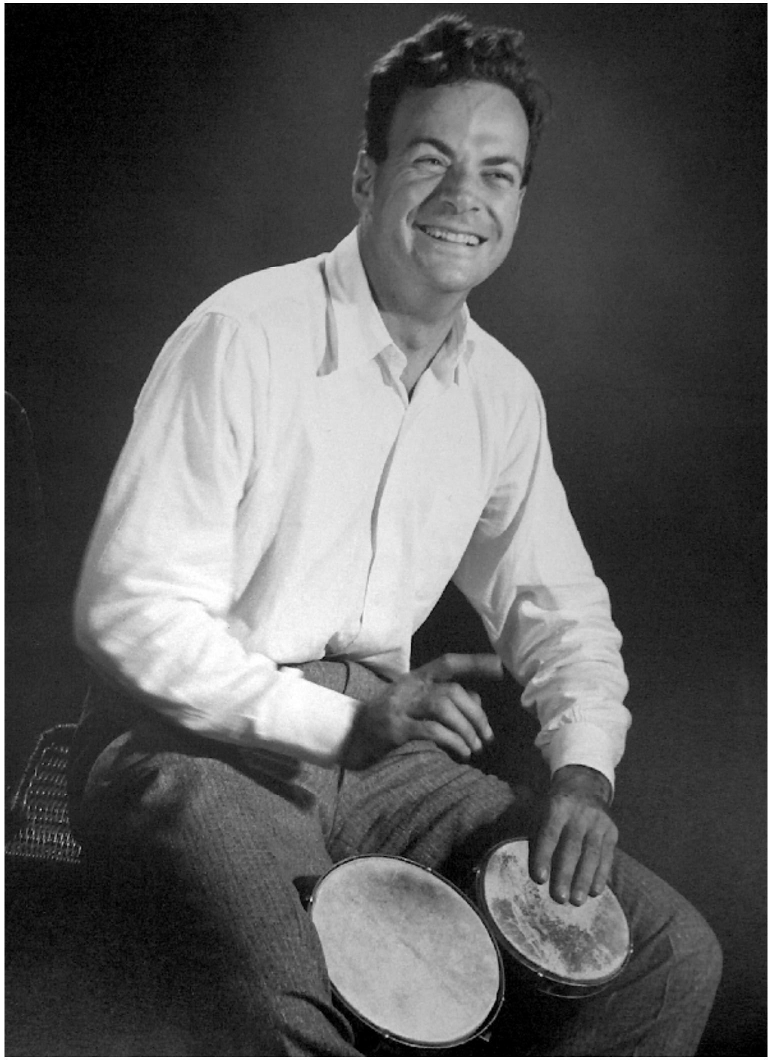
AND S. BENNETT, H. BARNETT, AND W. DEARDEN, NOTTINGHAM.

1828.

What Did George Green Look Like?

The Green's Function and Feynman Diagrams – *Propagators*





The Lippmann-Schwinger Equation

$$\psi(\mathbf{r},k) = \psi_i(\mathbf{r},k) + g(r,k) \otimes_3 V(\mathbf{r})\psi(\mathbf{r},k)$$

Incident + Scattered fields

- Forward scattering problem

$$Given\ V\ \ compute\ \psi$$

- Inverse scattering problem

$$Given\ \psi\ compute\ V$$

The Born Approximation

$$\psi(\mathbf{r},k) = \psi_i(\mathbf{r},k) + g(r,k) \otimes_3 V(\mathbf{r})\psi(\mathbf{r},k)$$

$$\psi_s \sim g(r,k) \otimes_3 V(\mathbf{r})\psi_i(\mathbf{r},k)$$

$$\|V(\mathbf{r})\| << 1$$

"I am now convinced that theoretical physics is actual philosophy"

Max Born

Far field Approximation

$$|\,\mathbf{r}-\mathbf{r}_0\,| = \sqrt{r_0^2 + r^2 - 2\mathbf{r}\cdot\mathbf{r}_0} = r_0\left(1 - \frac{2\mathbf{r}\cdot\mathbf{r}_0}{r_0^2} + \frac{r^2}{r_0^2}\right)^{\frac{1}{2}}$$

$$= r_0\left(1 - \frac{\mathbf{r}\cdot\mathbf{r}_0}{r_0^2} + \frac{r^2}{2r_0^2} + \ldots\right) \simeq r_0 - \hat{\mathbf{n}}_0\cdot\mathbf{r}$$

$$\hat{\mathbf{n}}_0 = \frac{\mathbf{r}_0}{r_0} \qquad \frac{r}{r_0} << 1$$

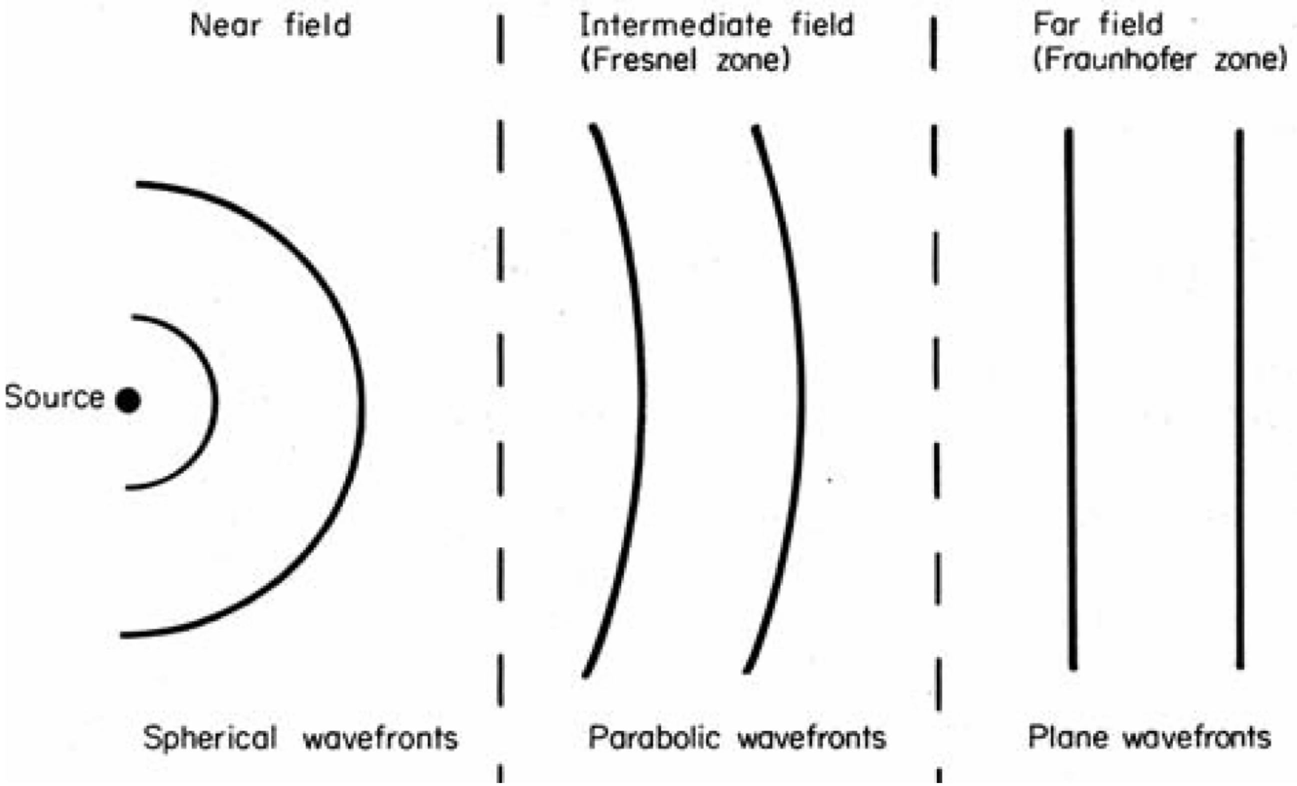
Farfield approximation

$$g(\mathbf{r}\mid\mathbf{r}_0,k) = \frac{1}{4\pi r_0}\exp(ikr_0)\exp(-ik\hat{\mathbf{n}}_0\cdot\mathbf{r})$$

Fresnel Approximation

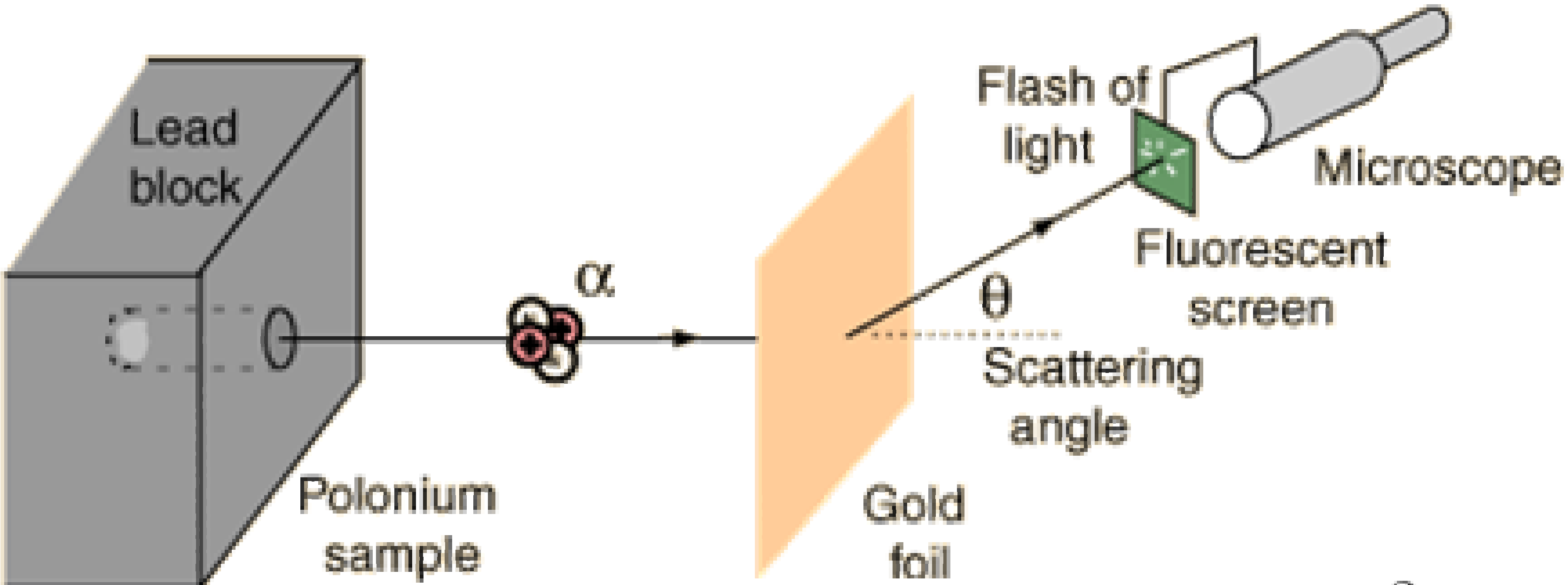
Based on including the ***quadratic phase factor***

$$g(\mathbf{r} \mid \mathbf{r}_0, k) = \frac{\exp(ikr_0)}{4\pi r_0} \exp(-ik\hat{\mathbf{n}}_0 \cdot \mathbf{r}) \exp(ir^2/2r_0)$$



Far Field Solution and Rutherford Scattering

$$\psi_s(\hat{\mathbf{n}}_i, \hat{\mathbf{n}}_s, k) \sim \int \exp[-ik(\hat{\mathbf{n}}_s - \hat{\mathbf{n}}_i) \cdot \mathbf{r}] V(\mathbf{r}) d^3\mathbf{r}$$



$$\|V(\mathbf{r})\| << 1$$

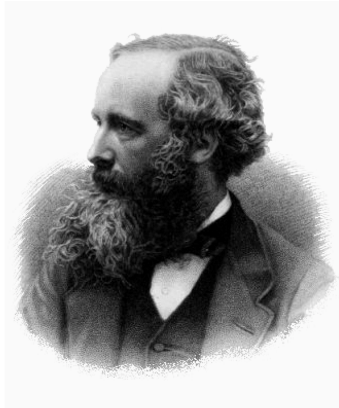
$$|\psi_s|^2 \propto \frac{1}{\sin^4 \frac{\theta}{2}}$$

What does the Born Approximation do for us?

Far field detection of waves is equivalent to **Fourier space** analysis of scatterer

Electromagnetic Scattering

- Consider Maxwell's equations for linear, isotropic but ***inhomogenous medium***



$$\nabla \cdot \epsilon \mathbf{E} = \rho,$$

$$\nabla \cdot \mu \mathbf{H} = 0,$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t},$$

$$\nabla \times \mathbf{H} = \epsilon \frac{\partial \mathbf{E}}{\partial t} + \mathbf{j}.$$

$$\mathbf{j} = \sigma \mathbf{E}$$

$$\rho(t) = \rho_0 \exp(-\sigma t/\epsilon)$$

$$\nabla \cdot \epsilon \mathbf{E} = 0$$

High conductivity condition

Wave Equation for the ***Electric Field***

- Decoupling Maxwell's equations for the Electric field (under the high conductivity condition) we have

$$\nabla^2 \mathbf{E} - \epsilon\mu \frac{\partial^2 \mathbf{E}}{\partial t^2} - \sigma\mu \frac{\partial \mathbf{E}}{\partial t} = -\nabla(\mathbf{E} \cdot \nabla \ln \epsilon) - (\nabla \ln \mu) \times \nabla \times \mathbf{E}$$

- In order to use a Green's function solution, we require this equation to be written in the form of the ***Langevin*** equation with a homogenous operator on the LHS

Langevin Form of the Wave Equation

$$\gamma_\epsilon = \frac{\epsilon - \epsilon_0}{\epsilon_0} \quad \text{and} \quad \gamma_\mu = \frac{\mu - \mu_0}{\mu}$$

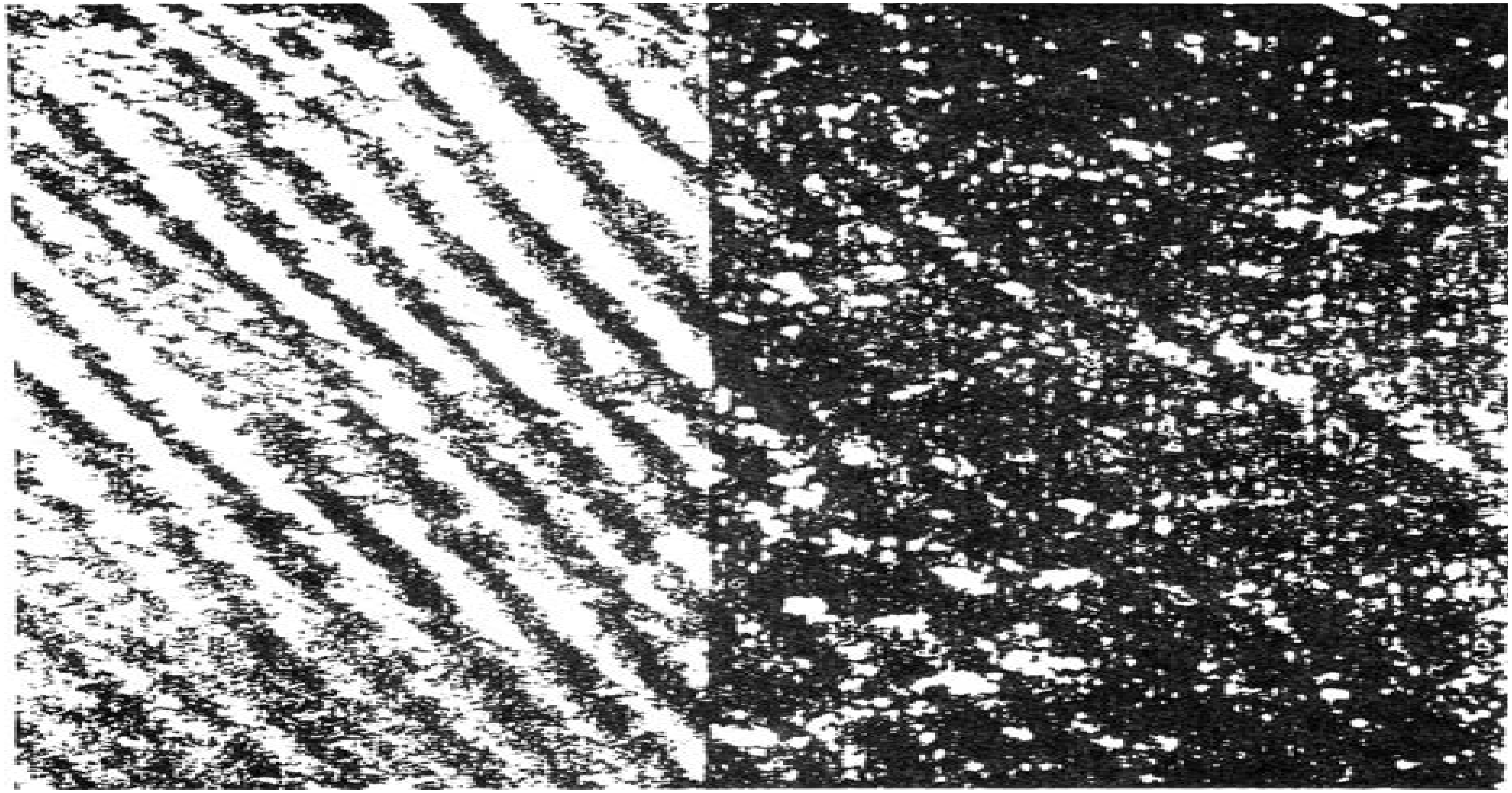
$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \widetilde{\mathbf{E}}(\mathbf{r}, \omega) \exp(i\omega t) d\omega$$

$$(\nabla^2 + k^2)\widetilde{\mathbf{E}} = -k^2 \gamma_\epsilon \widetilde{\mathbf{E}} + ikz_0\sigma\widetilde{\mathbf{E}} - \nabla(\widetilde{\mathbf{E}} \cdot \nabla \ln \epsilon) - \nabla \times (\gamma_\mu \nabla \times \widetilde{\mathbf{E}})$$

$$k = \frac{\omega}{c_0}, \quad c_0 = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \quad \text{and} \quad z_0 = \mu_0 c_0$$

Polarization Effects:
Real Aperture Radar

$$(\nabla^2 + k^2)\widetilde{\mathbf{E}}_s = -k^2\gamma\widetilde{\mathbf{E}}_i + ikz_0\sigma\widetilde{\mathbf{E}}_i - \nabla(\widetilde{\mathbf{E}}_i \cdot \nabla \ln \epsilon_r)$$



VV Polarization HH Polarization

Scalar EM Scattering Theory

- Based on the **scalar** ***Helmholtz equation***

$$(\nabla^2 + k^2)u(\mathbf{r}, k) = -k^2\gamma(\mathbf{r})u(\mathbf{r}, k)$$

$$\gamma(\mathbf{r}) \quad \exists \quad \forall \quad \mathbf{r} \in V$$

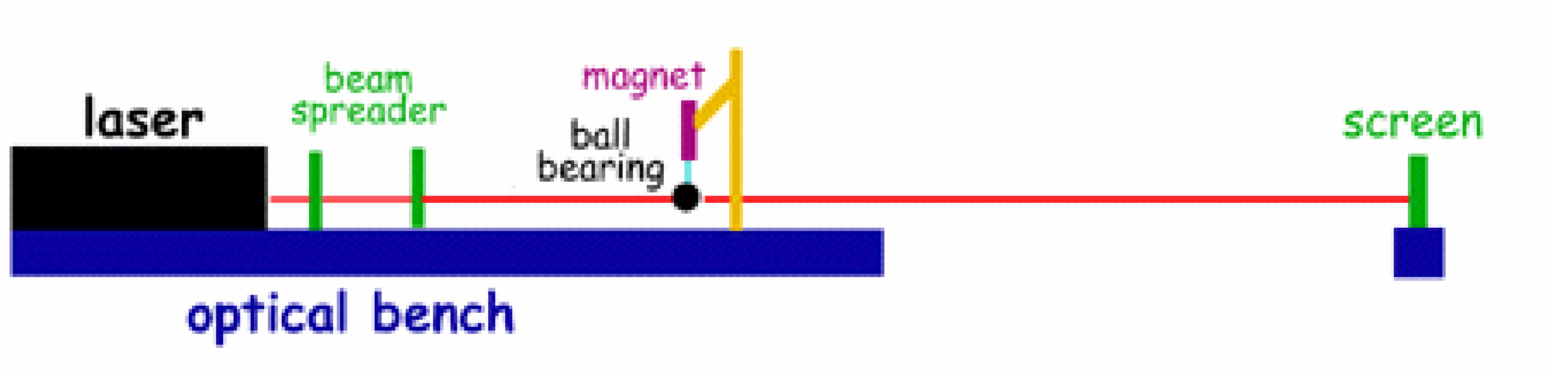
where V denotes volume

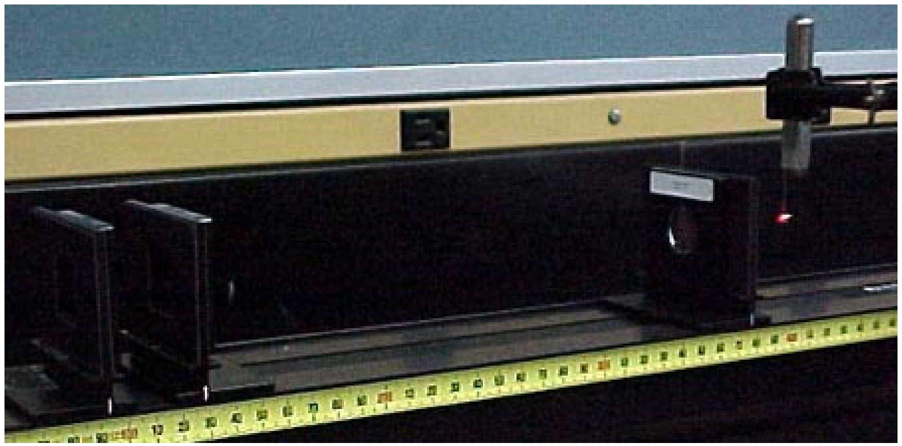
- Under the ***Born approximation*** $k^2\|\gamma(\mathbf{r})\| << 1$

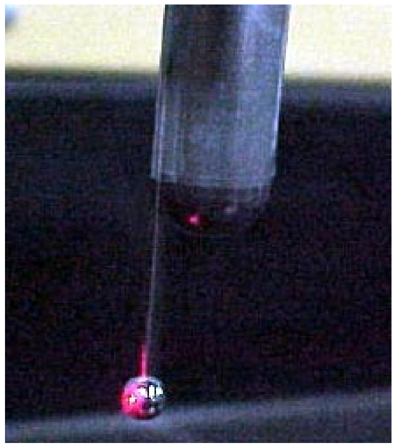
$$u_s(\mathbf{r}, k) = k^2 g(r, k) \otimes_3 \gamma(\mathbf{r})u(\mathbf{r}, k)$$

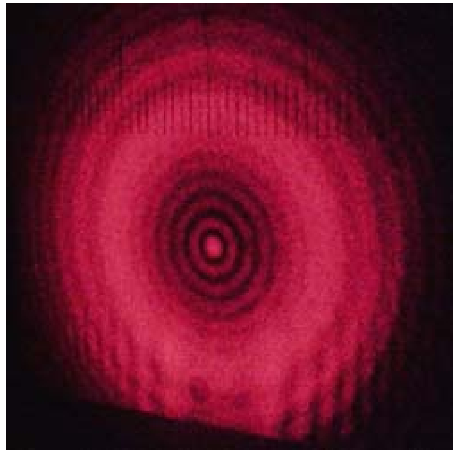
$$\sim k^2 g(r, k) \otimes_3 \gamma(\mathbf{r})u_i(\mathbf{r}, k)$$

Born Scattering and the *Poisson Spot*









Mathematical Model for the Poisson Spot (Diffraction)

$$u_s(x, y, z, k)$$

$$= k^2 \frac{\exp(ik\sqrt{x^2 + y^2 + z^2})}{4\pi\sqrt{x^2 + y^2 + z^2}} \otimes_3 \gamma(x, y)\delta(z) \exp(ikz)$$

$$= k^2 \frac{\exp(ik\sqrt{x^2 + y^2 + z^2}}{4\pi\sqrt{x^2 + y^2 + z^2}} \otimes_2 \gamma(x, y), \quad \gamma \quad \exists \quad \forall(x, y) \in S$$

Analysis

$$u_s(x_0, y_0, z_0, k)$$

$$= k^2 \int\int \frac{\exp[ik\sqrt{(x-x_0)^2+(y-y_0)^2+z_0^2}]}{4\pi\sqrt{(x-x_0)^2+(y-y_0)^2+z_0^2}}\gamma(x,y)dxdy$$

$$z_0\left(1+\frac{(x-x_0)^2}{z_0^2}+\frac{(y-y_0)^2}{z_0^2}\right)^{\frac{1}{2}}$$

$$\simeq z_0 - \frac{xx_0}{z_0} - \frac{yy_0}{z_0} + \frac{x_0^2}{2z_0} + \frac{y_0^2}{2z_0}$$

Analysis (Continued)

$$u_s(x_0, y_0, z_0, k) = \frac{\exp(ikz_0)}{4\pi z_0} \exp\left(ik\frac{x_0^2 + y_0^2}{2z_0}\right) A(u, v)$$

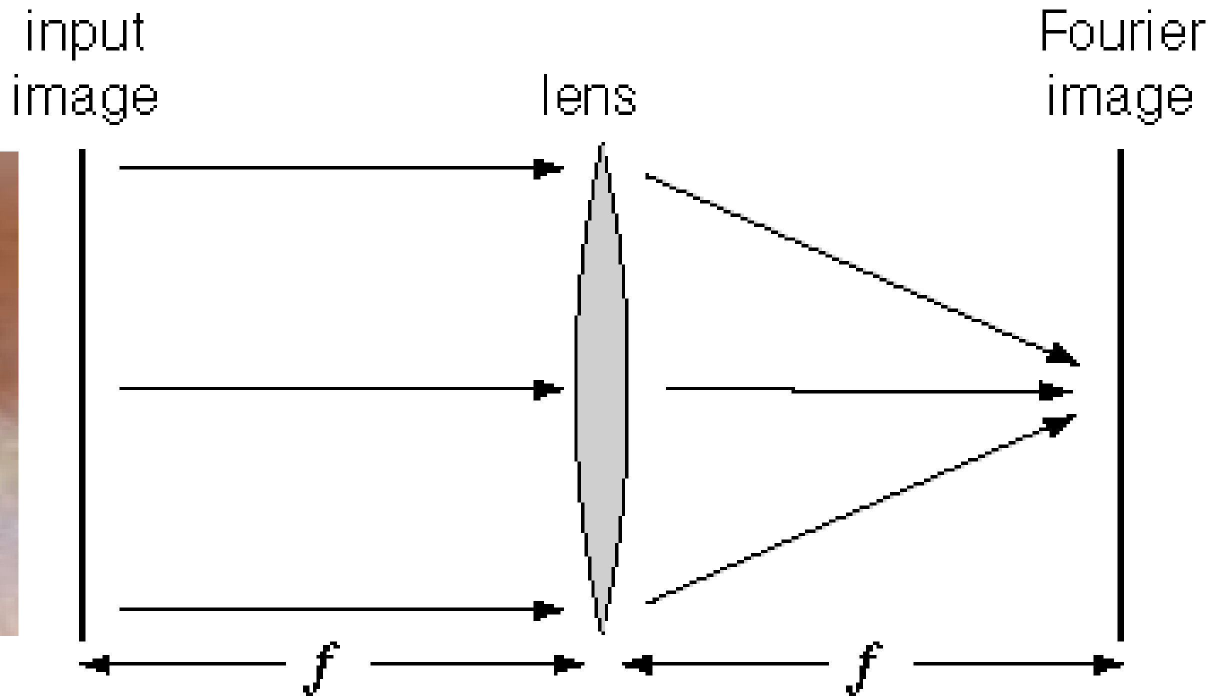
where

$$A(u, v) = k^2 \widetilde{\gamma}(u, v) = k^2 \mathcal{F}_2[\gamma(x, y)]$$

$$= k^2 \int\int \exp(-iux) \exp(-ivy) \gamma(x, y) dx dy$$

$$u = \frac{kx_0}{z_0} = \frac{2\pi x_0}{\lambda z_0} \qquad v = \frac{ky_0}{z_0} = \frac{2\pi y_0}{\lambda z_0}$$

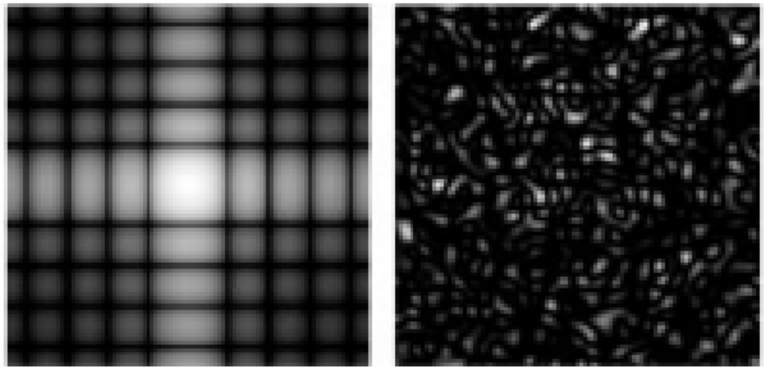
An Inverse (Born) Scattering Process we are doing now



Coherent and Incoherent Images

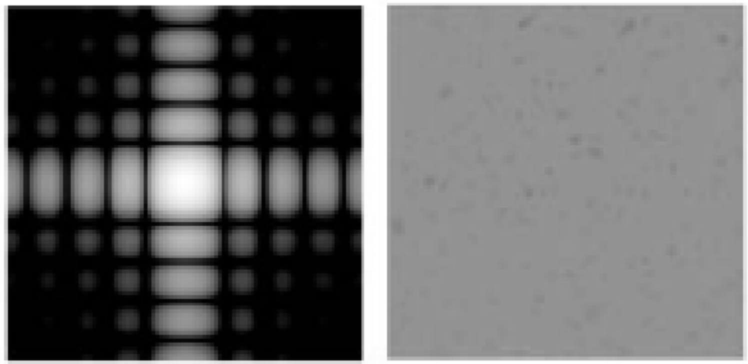
- Model for a coherent image

$$I = | p \otimes\otimes f + n |^2$$



- Model for an incoherent image

$$I = | p |^2 \otimes\otimes | f |^2 + | n |^2$$



What's Wrong with the Born Approximation ?

$$k^2 \|\gamma(\mathbf{r})\| << 1$$

- Translates to:

$$\lambda >> \mathrm{Scatterer}$$

- Information based on

$$\lambda \sim \mathrm{Scatterer}$$

Strong Scattering Model

$$\text{wavefield generated by single scattering events}$$
$$+$$
$$\text{wavefield generated by double scattering events}$$
$$+$$
$$\text{wavefield generated by triple scattering events}$$
$$+$$
$$\vdots$$

The Born Series and Noise

$$u = u_i + k^2 g \otimes_3 \gamma u$$

$$= u_i + k^2 g \otimes_3 \gamma u_i + k^4 g \otimes_3 \gamma (g \otimes_3 \gamma u_i) + \ldots$$

$$u_s = k^2 g \otimes_3 \gamma u_i + n$$

Signal = IRF convolved **Input + Noise**

The ***noise term*** describes
multiple scattering processes

Accuracy of the Model

Does not take into account higher order effects

double, triple, …   scattering effects

Other Methods

- Based on an Eikonal transformation of the type

$$u = u_i \exp(s)$$

- Under the Rytov approximation we obtain

$$u(\mathbf{r}, k) = u_i(r\, k) \exp\left[\frac{k^2 g(r,k) \otimes_3 \gamma(\mathbf{r}) u_i(\mathbf{r}, k)}{u_i(\mathbf{r}, k)}\right]$$

$$\|k^2\gamma\| >> \|\nabla s \cdot \nabla s\|$$

Inverse Solution Method

Inverse Scattering Solutions with Applications to Electromagnetic Signal Processing, Blackledge et al;
ISAST Journal of Electronics and Signal Processing,
Vol. 4, No. 1, 43 - 60, 2009; http://eleceng.dit.ie/papers/113.pdf

Theorem: If $$(\nabla^2 + k^2)u(\mathbf{r}, k) = -k^2\gamma(\mathbf{r})u(\mathbf{r}, k)$$

and

$$u(\mathbf{r}, k) = u_i(\mathbf{r}, k) + u_s(\mathbf{r}, k) \qquad (\nabla^2 + k^2)u_i = 0$$

then

$$\gamma(\mathbf{r}) = \frac{u^*(\mathbf{r}, k)}{\mid u(\mathbf{r}, k) \mid^2}\nabla^2\left[\frac{1}{4\pi r} \otimes_3 u_s(\mathbf{r}, k) - \frac{1}{k^2}u_s(\mathbf{r}, k)\right]$$

Proof

$$
\begin{aligned}
\gamma &= \frac{1}{u}\nabla^2\left[\frac{1}{4\pi r}\otimes_3(u-u_i)-\frac{1}{k^2}(u-u_i)\right]\\
&= \frac{1}{u}\left[-\delta^3\otimes_3(u-u_i)-\frac{1}{k^2}(\nabla^2 u-\nabla^2 u_i)\right]\\
&= \frac{1}{u}\left[-(u-u_i)-\frac{1}{k^2}\nabla^2 u+\frac{1}{k^2}\nabla^2 u_i\right]\\
&= \frac{1}{k^2 u}\left[-(\nabla^2 u+k^2 u)+\nabla^2 u_i+k^2 u_i\right]\\
&= -\frac{1}{k^2 u}(\nabla^2+k^2)u.
\end{aligned}
$$

Analysis 1:
Asymptotic Result

$$-k^2\gamma(\mathbf{r}) = \frac{u^*(\mathbf{r},k)}{\mid u(\mathbf{r},k)\mid^2}\nabla^2\left(u_s(\mathbf{r},k) - \frac{k^2}{4\pi r}\otimes_3 u_s(\mathbf{r},k)\right)$$

$$\|u_s - (k^2/4\pi r)\otimes_3 u_s\|_2 \leq \|u_s\|_2[1 + k^2\sqrt{r/(4\pi)}]$$

$$-k^2\gamma = \frac{-1}{u_i^{\pm} + u_s}k^2 u_s \otimes_3 \nabla^2\left(\frac{1}{4\pi r}\right)$$

$$= k^2 A^{-1}[(u_i^{\pm})^* + u_s^*]u_s, \quad r \to \infty$$

$$A^{-1} = \mid u_i^{\pm} + u_s \mid^{-2}$$

Analysis 2:

Transformation to Fourier Space

$$\widetilde{\gamma}(k\hat{\mathbf{n}}) = [\widetilde{u}_s[k(\hat{\mathbf{n}}_i - \hat{\mathbf{n}})] + \widetilde{u}_s^*(k\hat{\mathbf{n}}) \otimes_3 \widetilde{u}_s(k\hat{\mathbf{n}})] \otimes_3 \widetilde{A}^{-1}(k\hat{\mathbf{n}})$$

$$\widetilde{A}^{-1} = \delta^3, \quad \hat{\mathbf{n}}_i - \hat{\mathbf{n}} = \hat{\mathbf{n}}_s$$

$$\widetilde{u}_s(k\hat{\mathbf{n}}_s) = \widetilde{\gamma}[k(\hat{\mathbf{n}}_i - \hat{\mathbf{n}}_s)] - \widetilde{u}_s^*[k(\hat{\mathbf{n}}_i - \hat{\mathbf{n}}_s)] \otimes_3 \widetilde{u}_s[k(\hat{\mathbf{n}}_i - \hat{\mathbf{n}}_s)]$$

Scattered Field = Single Scattering - Multiple Scattering

Fourier zone Fourier transform Convolution

Summary

- Imaging systems modelling is based on using a ***Green's function solution*** to the wave equation best ***models the system***

- Application of the ***Born approximation*** provides a ***linear system theory*** approach to modelling an image

- ***Multiple scattering*** effects are taken to contribute to the ***additive noise term***

In the Following Lecture…

We shall consider a case study based on the following question:

Why are Einstein Rings Blue ?

Questions

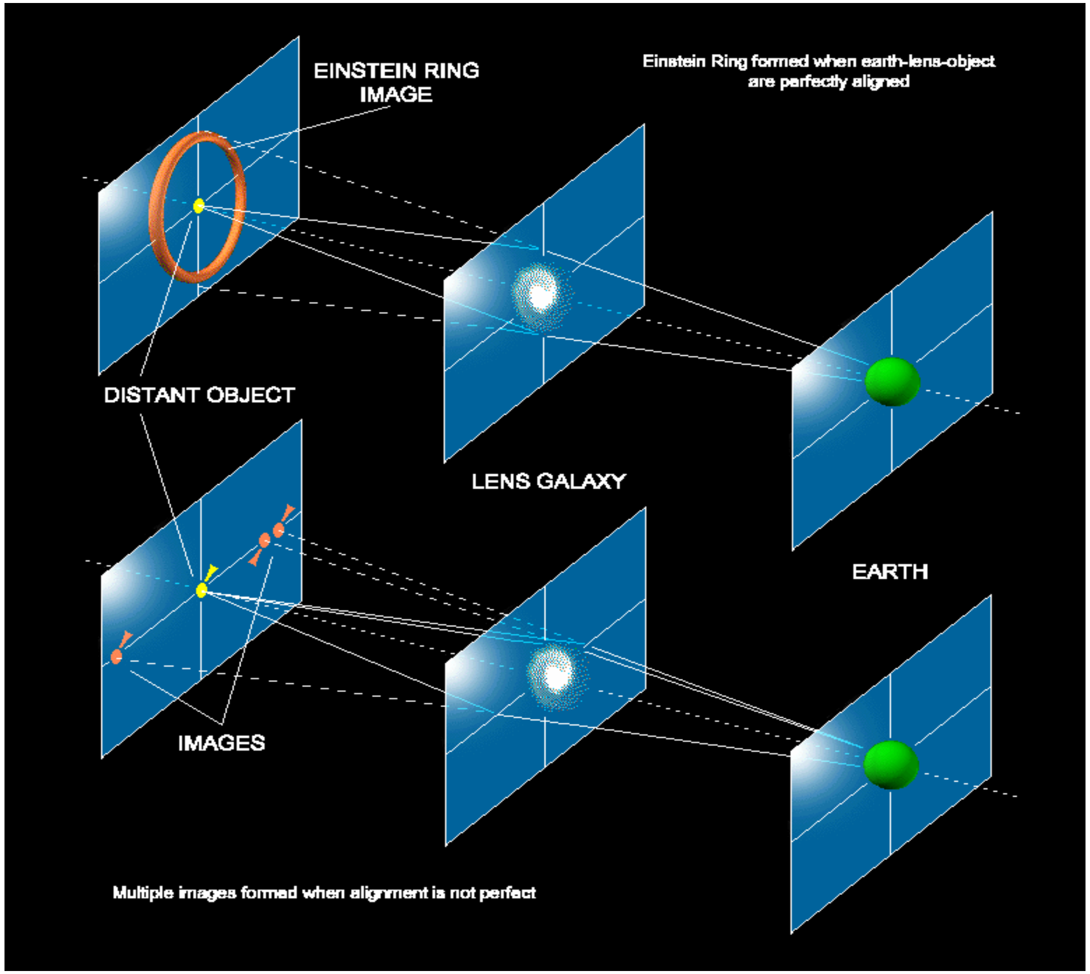
+

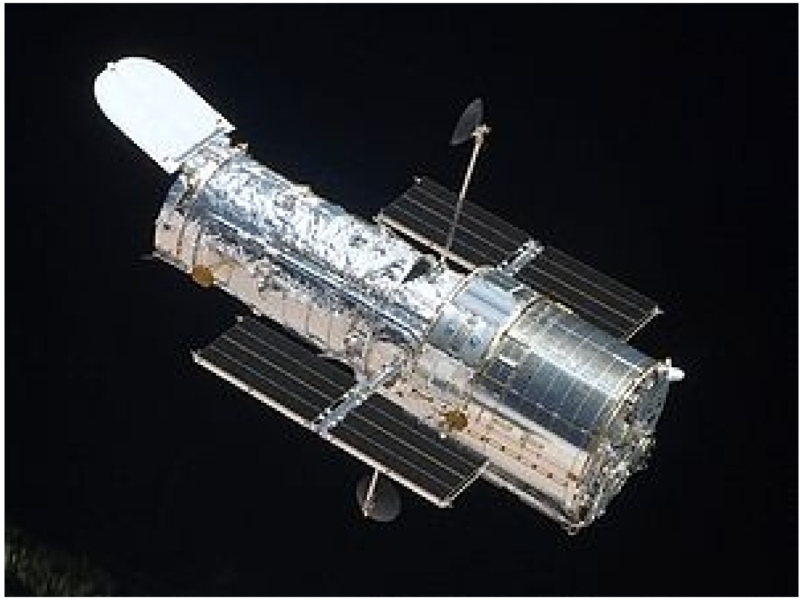
Interval (10 Minutes)

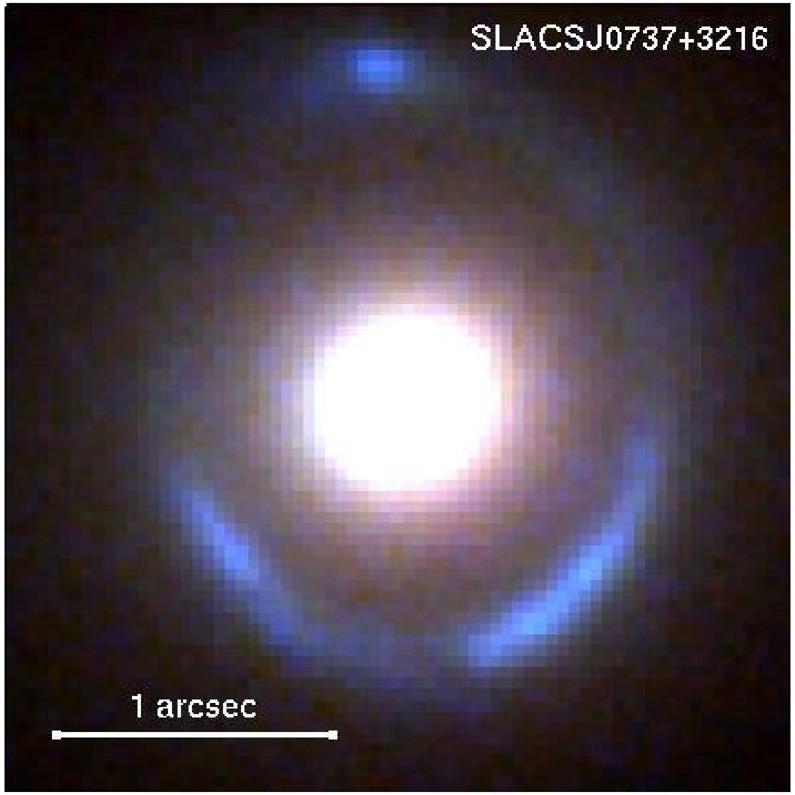
Part II: Contents

- The Hubble Space Telescope and Einstein rings
- Low frequency scattering theory
- Why is an Einstein ring blue?
- Compatibility with General Relativity
- What is Gravity?
- The field equations of physics
- The Maxwell-Proca equations
- Summary
- Q & A

The HST & Einstein Rings







Low Frequency Scattering

- Consider the wave equation

$$(\nabla^2 + k^2)\psi(\mathbf{r}, k) = -k^2\gamma(\mathbf{r})\psi(\mathbf{r}, k)$$

$$\gamma(\mathbf{r}) = \frac{2mc^2}{E^2}[E - E_p(\mathbf{r})] - 1$$

- Exact scattering solution is

$$\psi_s^0 = \lim_{k \to 0} \psi_s = \frac{k_0^2}{4\pi r} \otimes_3 \gamma$$

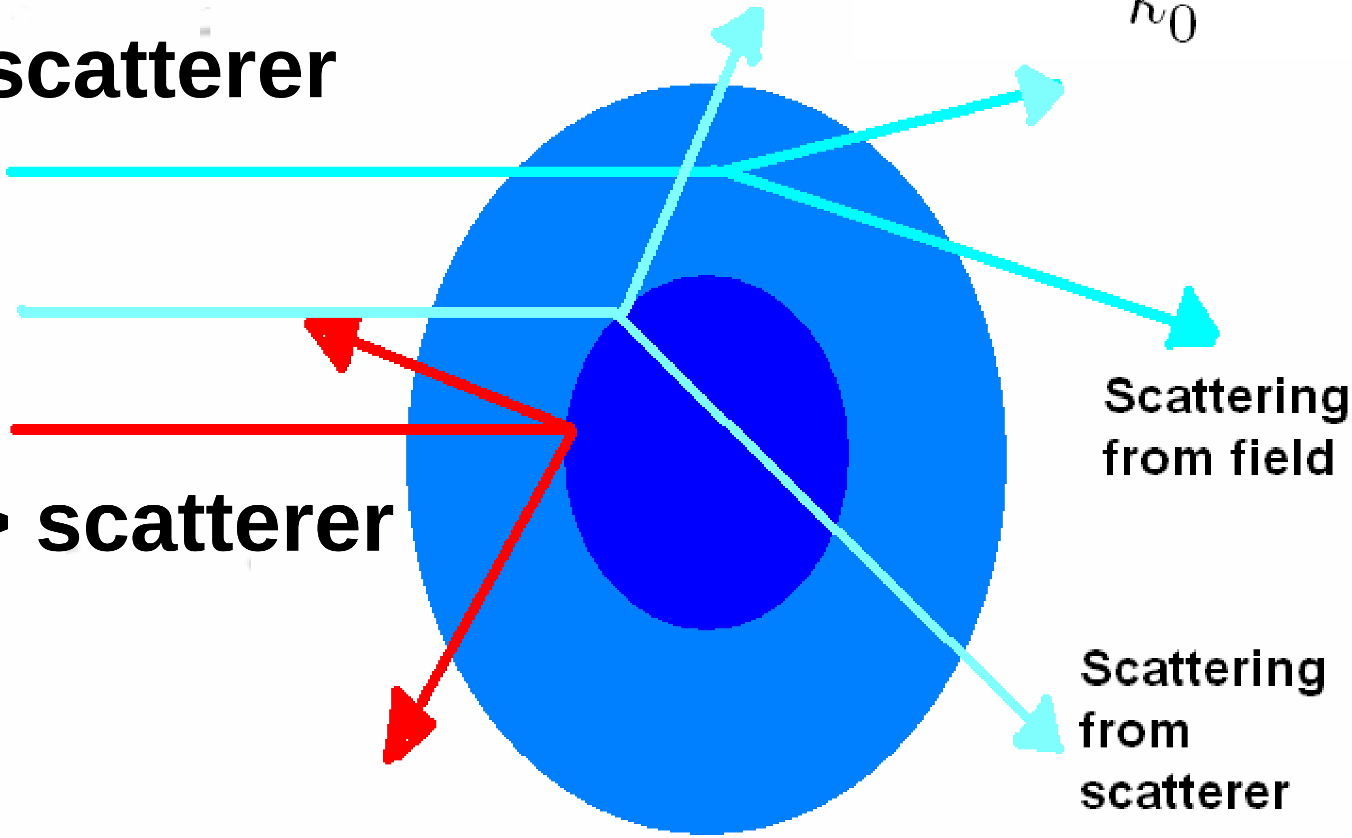
which is a general solution of

$$\nabla^2 \psi_s^0 = -k_0^2 \gamma$$

Scattering from a Low Frequency Scattered Field

$$\psi_s \sim -\frac{k^2}{k_0^2} g \otimes_3 \psi_i \nabla^2 \psi_s^0$$

l ~ **scatterer**



l **>> scatterer**

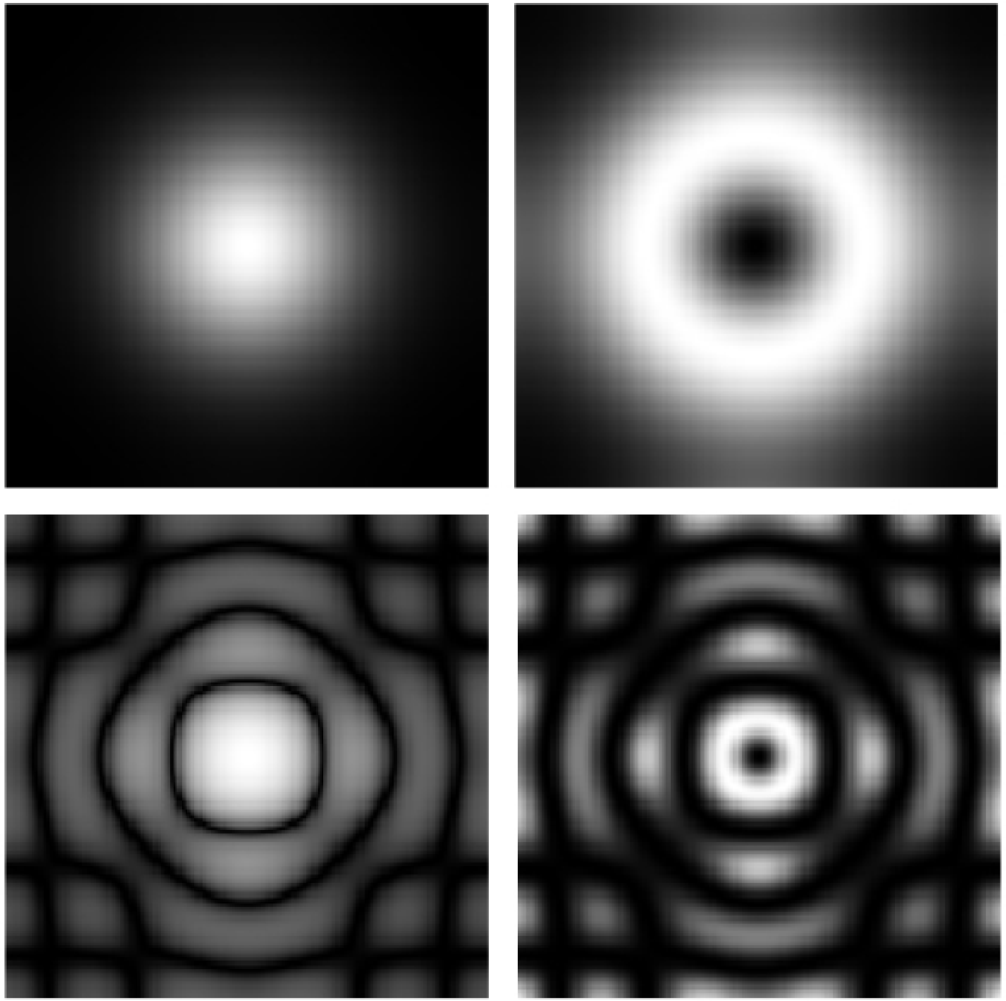
$$\nabla^2 \psi_s^0 = -k_0^2 \gamma$$

Field generated by low frequency scattering

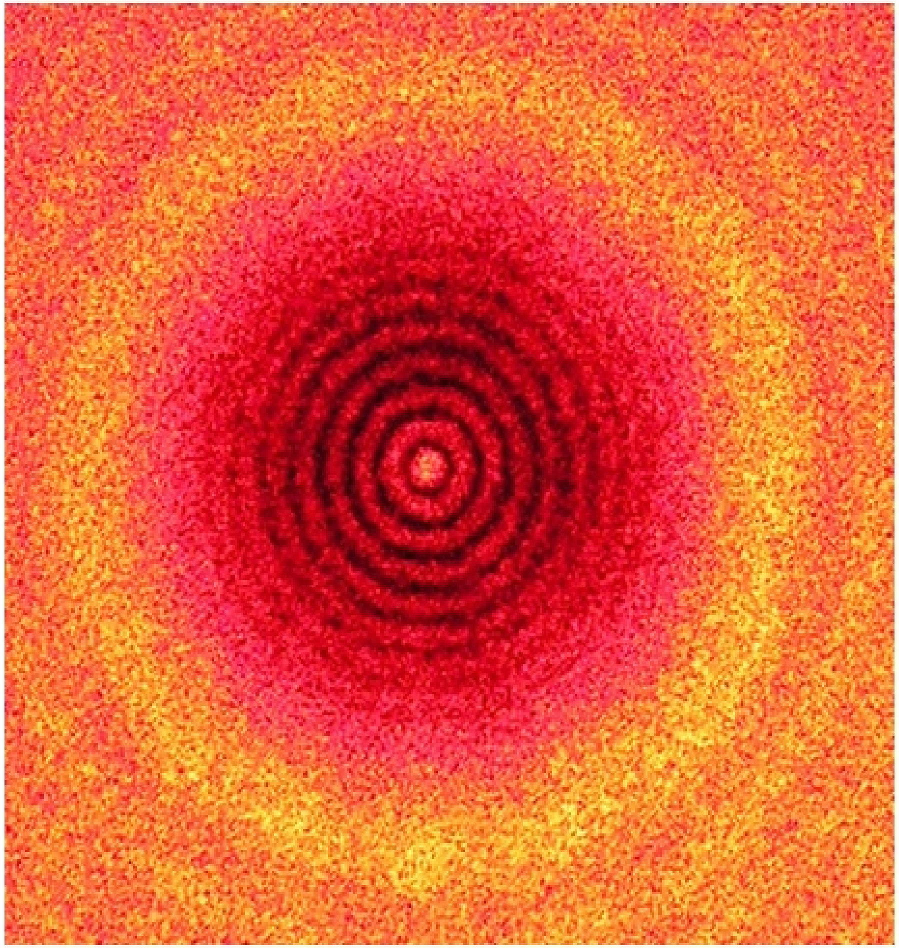
$$\psi_s \sim k^2 g \otimes_3 \gamma \psi_i$$

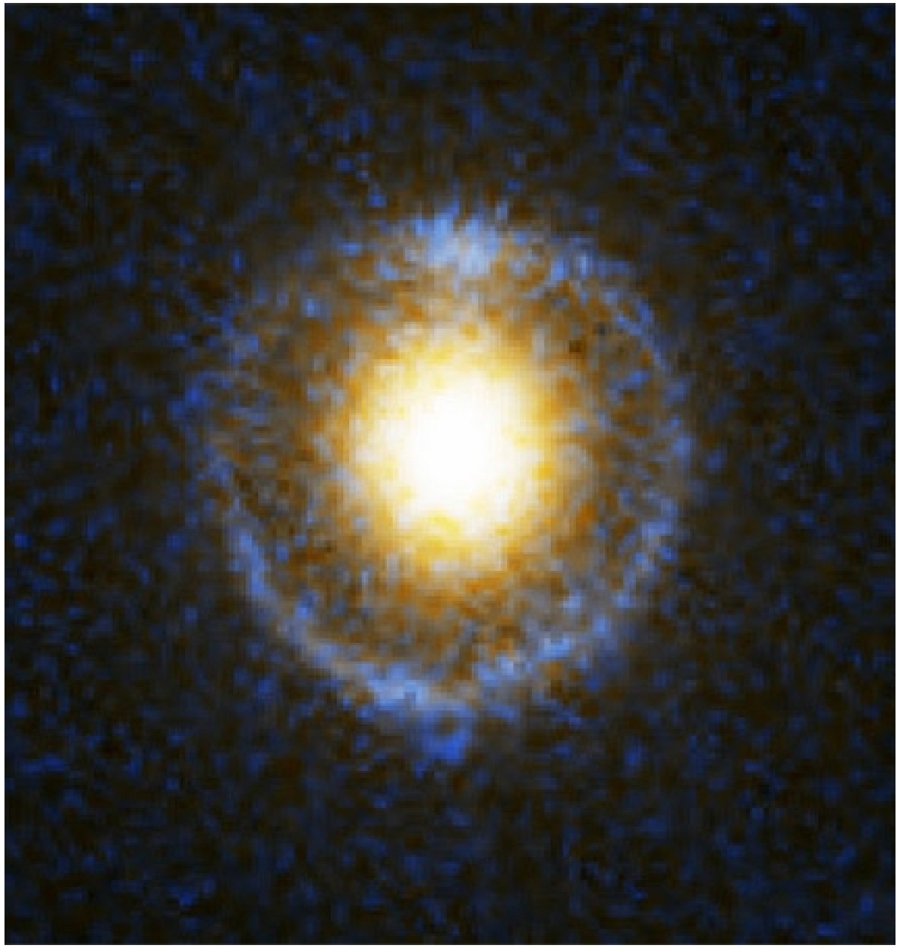
Far Field Diffraction Patterns

$$\psi_s \sim k^2 g \otimes_3 \gamma \psi_i \qquad \psi_s \sim -\frac{k^2}{k_0^2} g \otimes_3 \psi_i \nabla^2 \psi_s^0$$

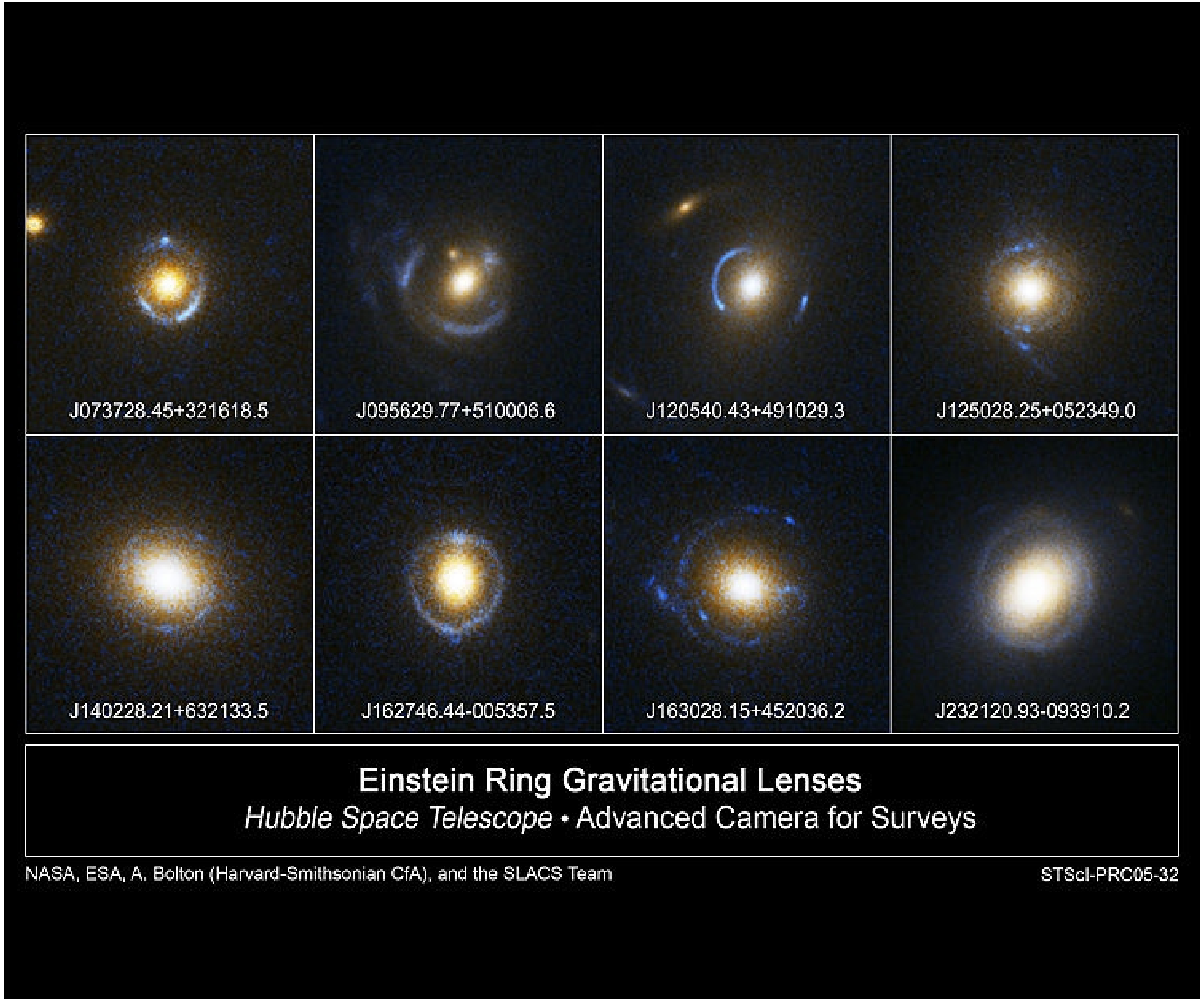


Poisson Spot .v. Einstein Ring

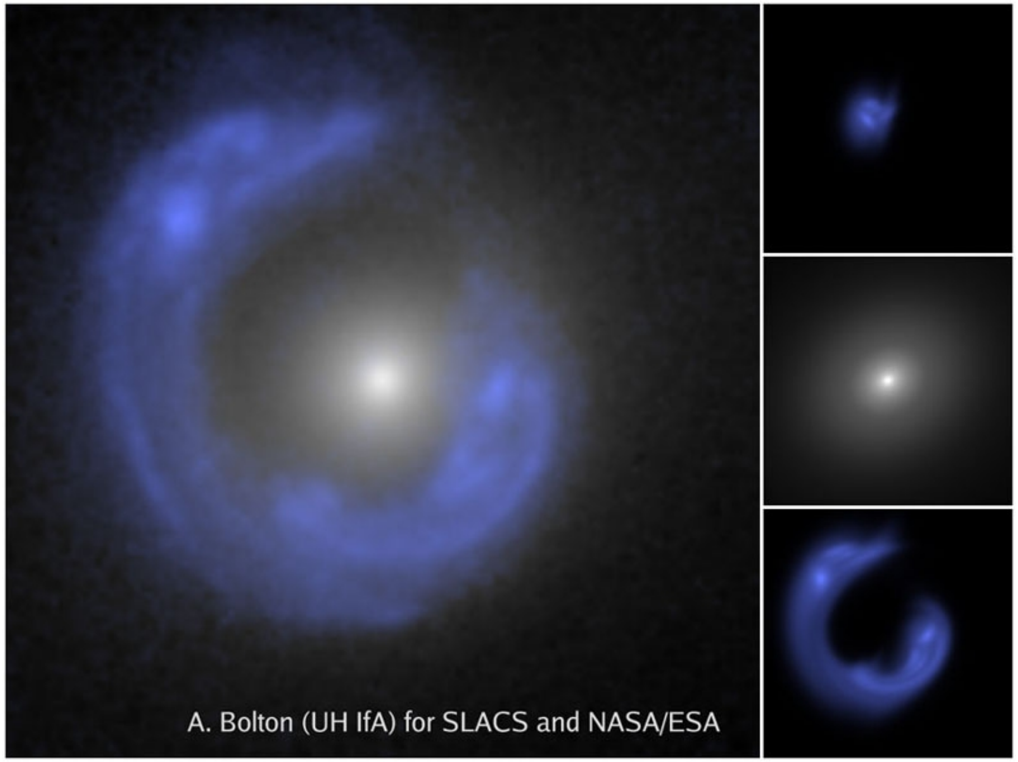




Why is an Einstein Ring Blue ?



A Dummkopf Explanation



“What's large and blue and can wrap itself around an entire galaxy? A gravitation lens image. Pictured above on the left, the gravity of a normal white galaxy has gravitationally distorted the light from a much more distant blue galaxy”.

Astronomy Picture of the Day, July 28, 2008

Scaling Laws

- Tyndall scattering of light

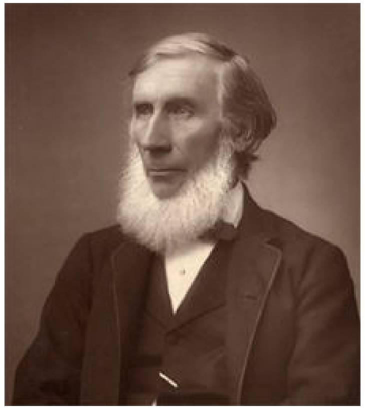
$$| \psi_s |^2 \propto \frac{1}{\lambda^2}$$

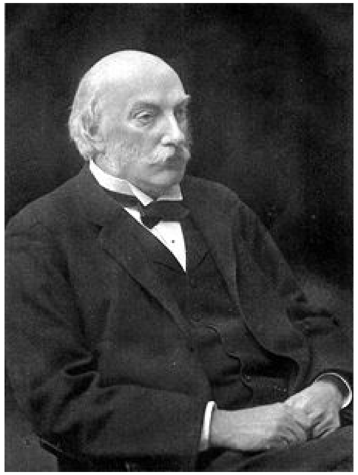
- Rayleigh scattering of light

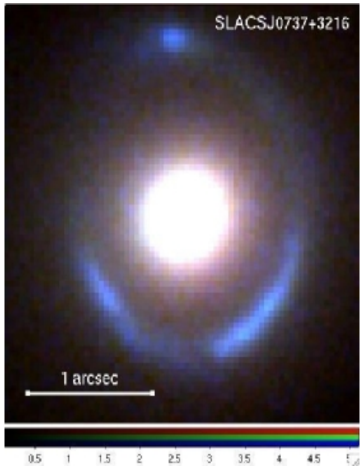
$$| \psi_s |^2 \propto \frac{1}{\lambda^4}$$

- ***Gravitational scattering of light?***

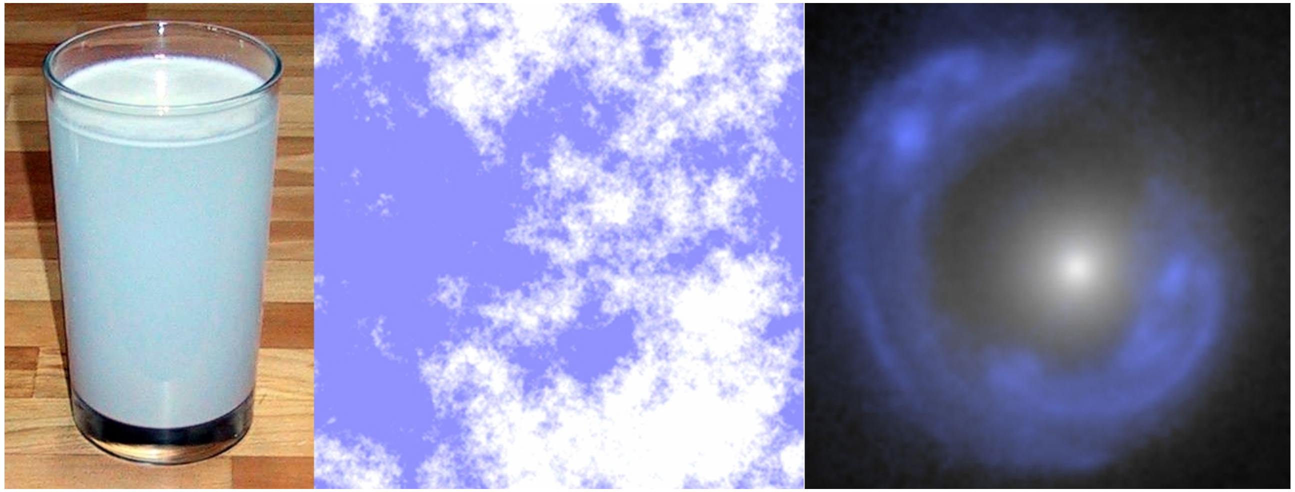
$$| \psi_s |^2 \propto \frac{1}{\lambda^6}$$







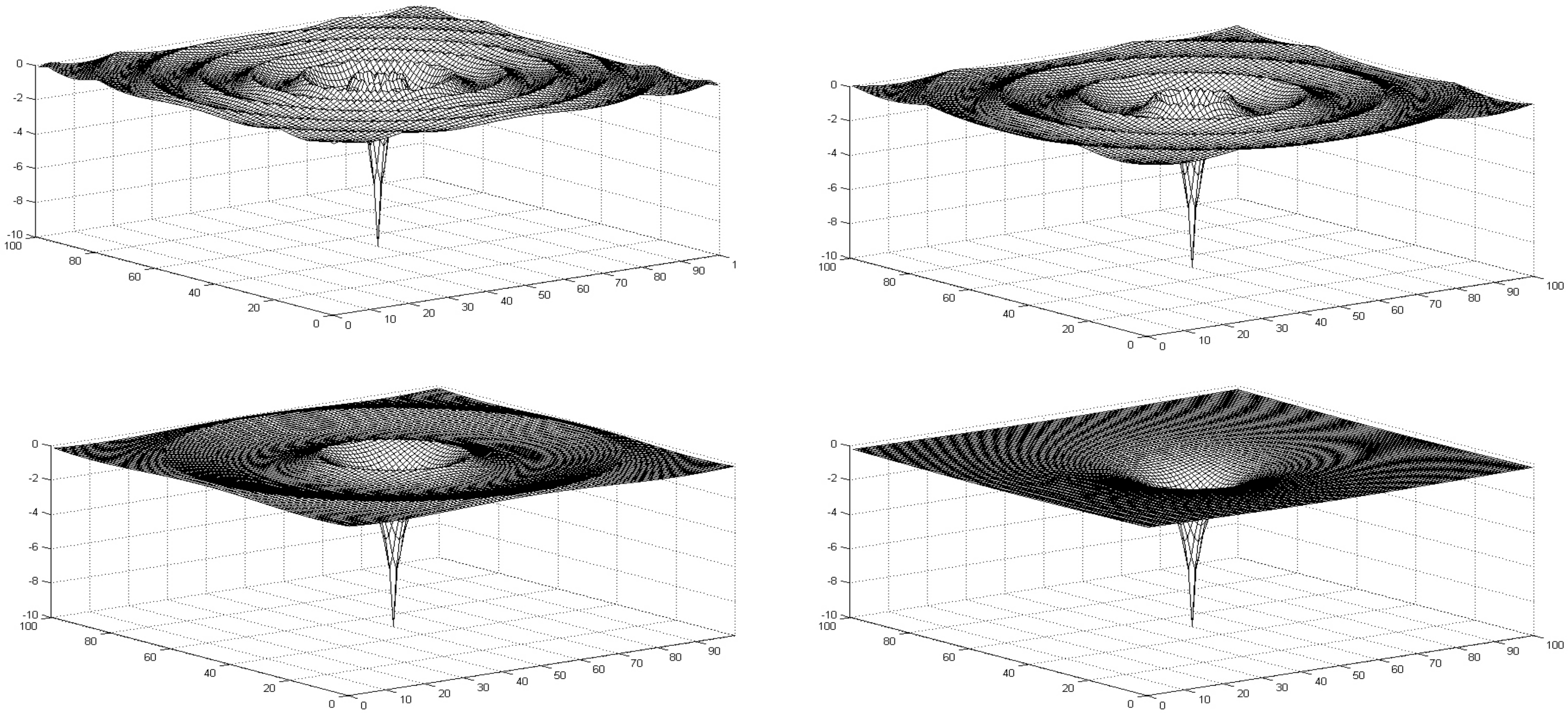
Experimental Evidence:
The Colour of Scattering



$-2\ln\lambda$	$-4\ln\lambda$	$-6\ln\lambda$

Compatibility with GR

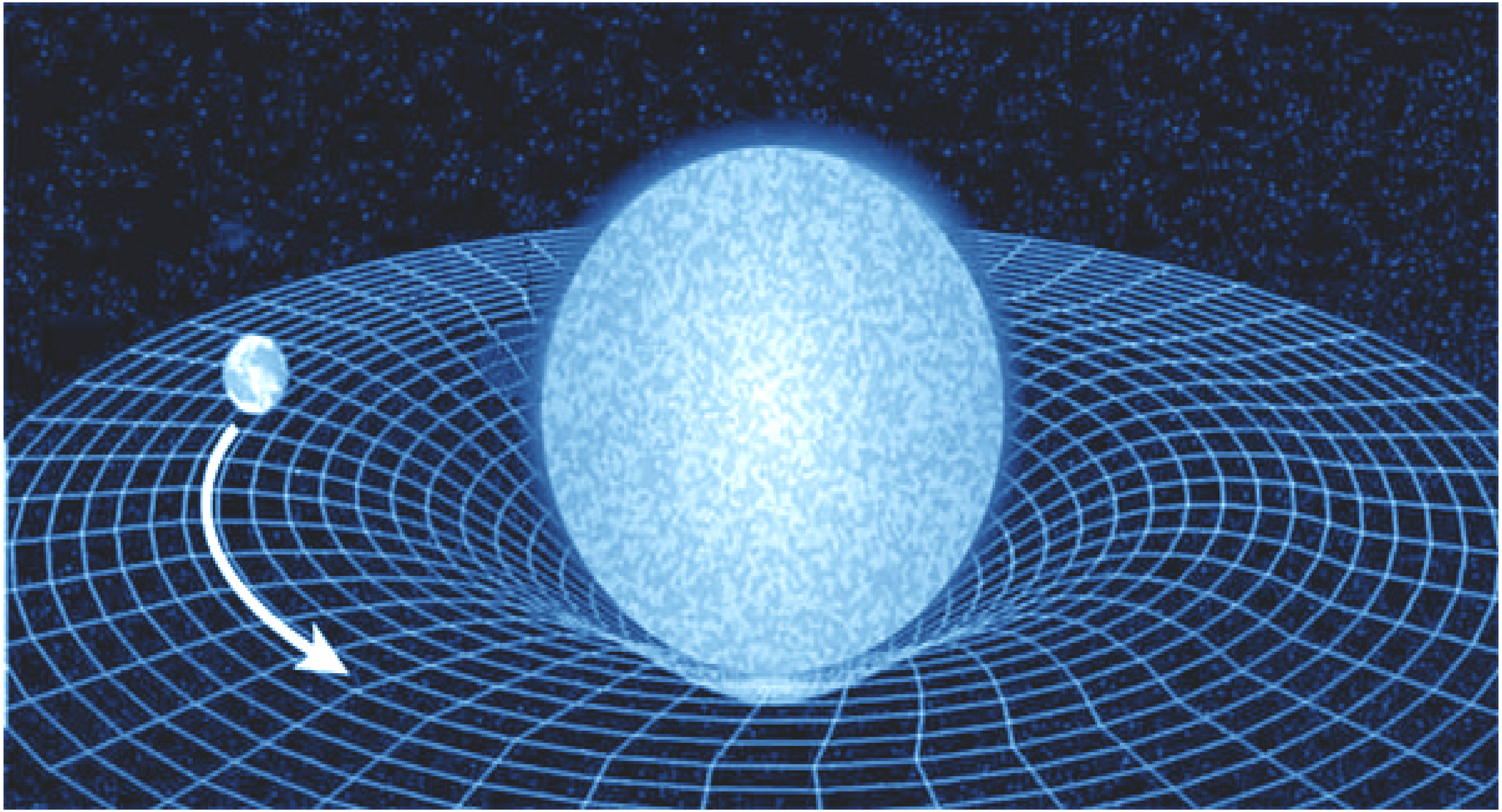
Two Dimensional 'Space Waves'



Let the 'medium' of wave propagation be

Space-Time

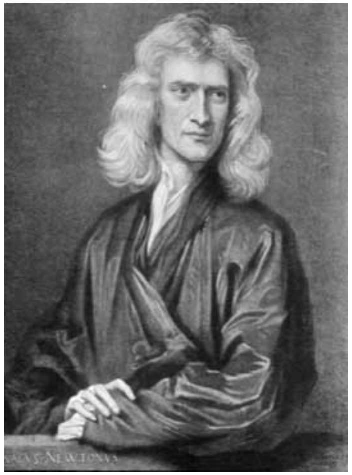
Curved Space Explanation of Gravity



What is Gravity ?

Two masses experience a gravitational force because each mass *'detects'* the 'low frequency wavefields' (gravity waves)

scattered by the



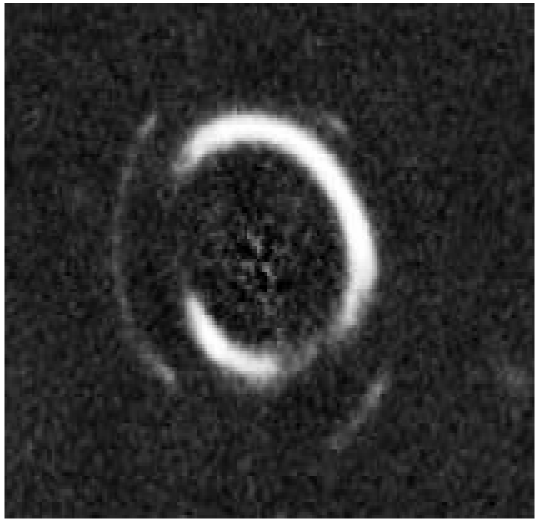
'high frequency wavefields' (matter waves) of the other.

Example Consequences

- Gravity waves (as predicted by Einstein) will not be measured because the detectors are in effect weighing machines designed to weigh themselves!

- A black hole is a 'strong scatterer' of gravity waves

- A black hole will generate multiple Einstein rings



The Field Equations of Physics

- Maxwell's equations (1865)
  Electromagnetic waves

- Einstein's equations (1916)
  Gravity waves

- Schrodinger (1925), Klein-Gordon (1927), Dirac (1928) equations (& others)
  Matter waves

Fields .v. Wavefields

- Unified field theory: Fields determine wavefields

  e.g. Maxwell's equations decouple to give the classical (non-relativistic) wave equation

  Fields describe mass-less Vector Bosons

- Unified wavefield theory: Wavefields determine fields

  e.g. Proca equations are Maxwell's equations designed specifically so that upon decoupling, the Klein-Gordon (relativistic) wave equation is obtained.

  Fields describe massive Vector Bosons

The Proca-Maxwell Equations

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)U - \kappa^2 U = f, \quad \kappa = \frac{mc}{\hbar}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)U - \kappa^2 U = -\frac{\rho}{\epsilon_0}, \qquad \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A} - \kappa^2 \mathbf{A} = -\mu_0 \mathbf{J}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} - \kappa^2 U, \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} - \kappa^2 \mathbf{A}$$

The Search for Unity in Physical Law: String Theory .v. Wave Theory

- String theory *('find the fundamental building blocks')*

  All physics is the result of waves or 'strings' with wavelengths ~ Planck length

$$\ell = \sqrt{\frac{\hbar G}{c^3}} \simeq 1.16 \times 10^{-35}\mathrm{m}$$

- Fractal wave theory *('waves within waves approach')*

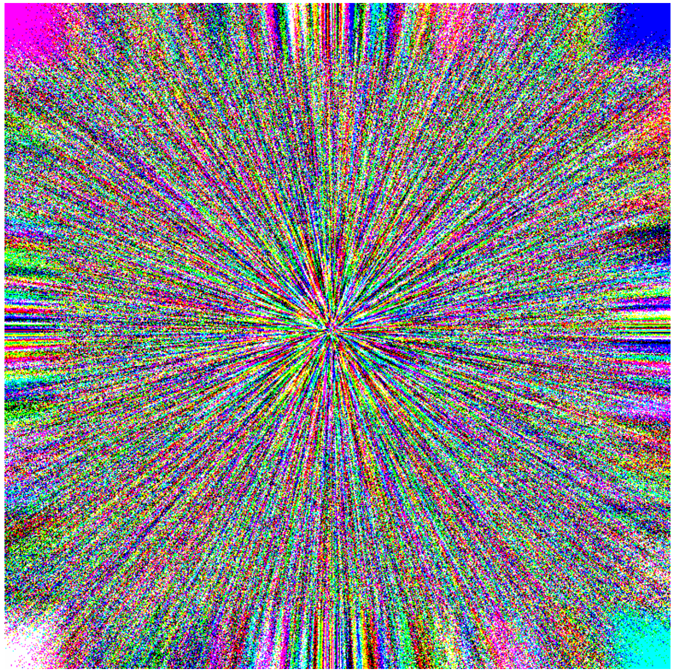
  All physics is the result of waves interacting (scattering) with waves at all wavelengths greater than the Planck length subject to the principle of (scale variant) eigenfield evolution

What Determines the Bandwidth

Big Bang:

short impulse

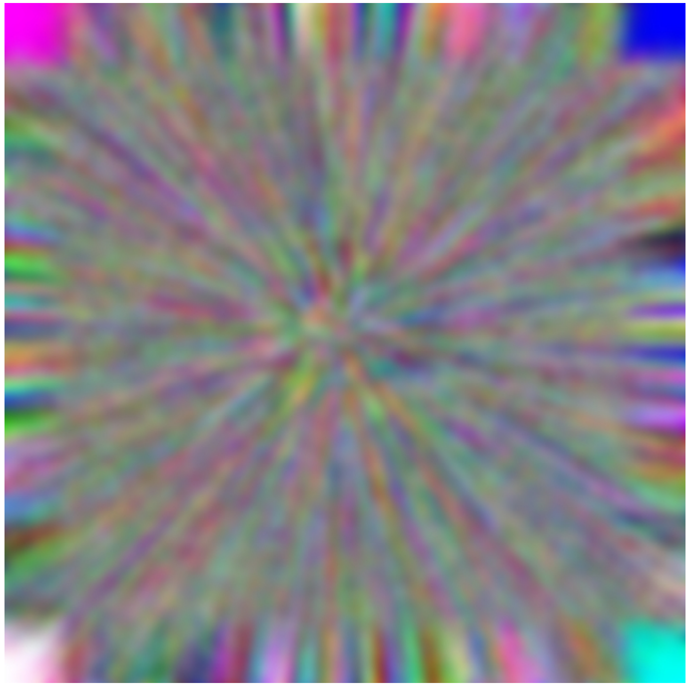
broad spectrum



Big Puff:

long impulse

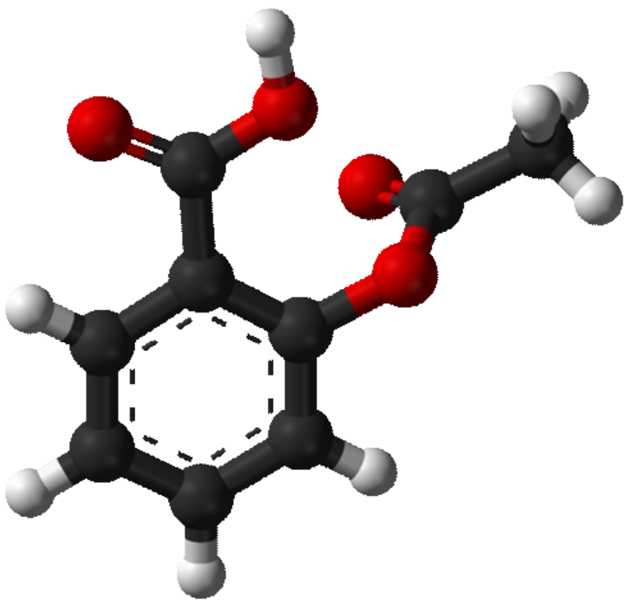
narrow spectrum

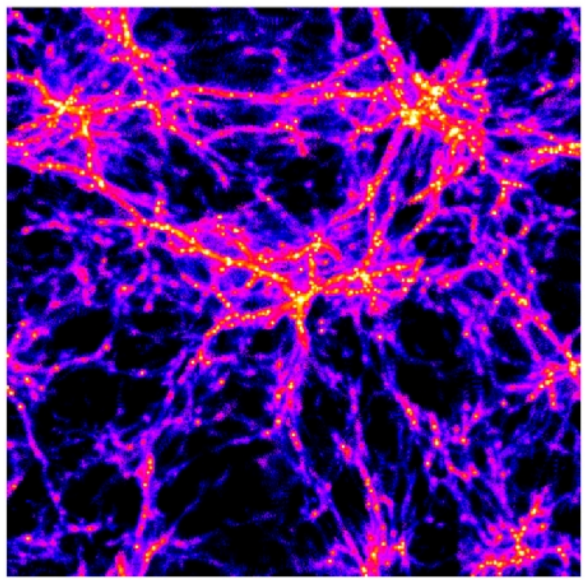


Eigenfield Evolution

Eigenfields evolve over a time period determined by scale and light speed

$$\text{Free Wavefield} \quad \underset{\text{Magnetic field } \frac{\partial \mathbf{E}}{\partial t}}{\overset{\text{Electric Field } \mathbf{E}}{\longrightarrow}} \quad \text{Eigen Wavefield}$$

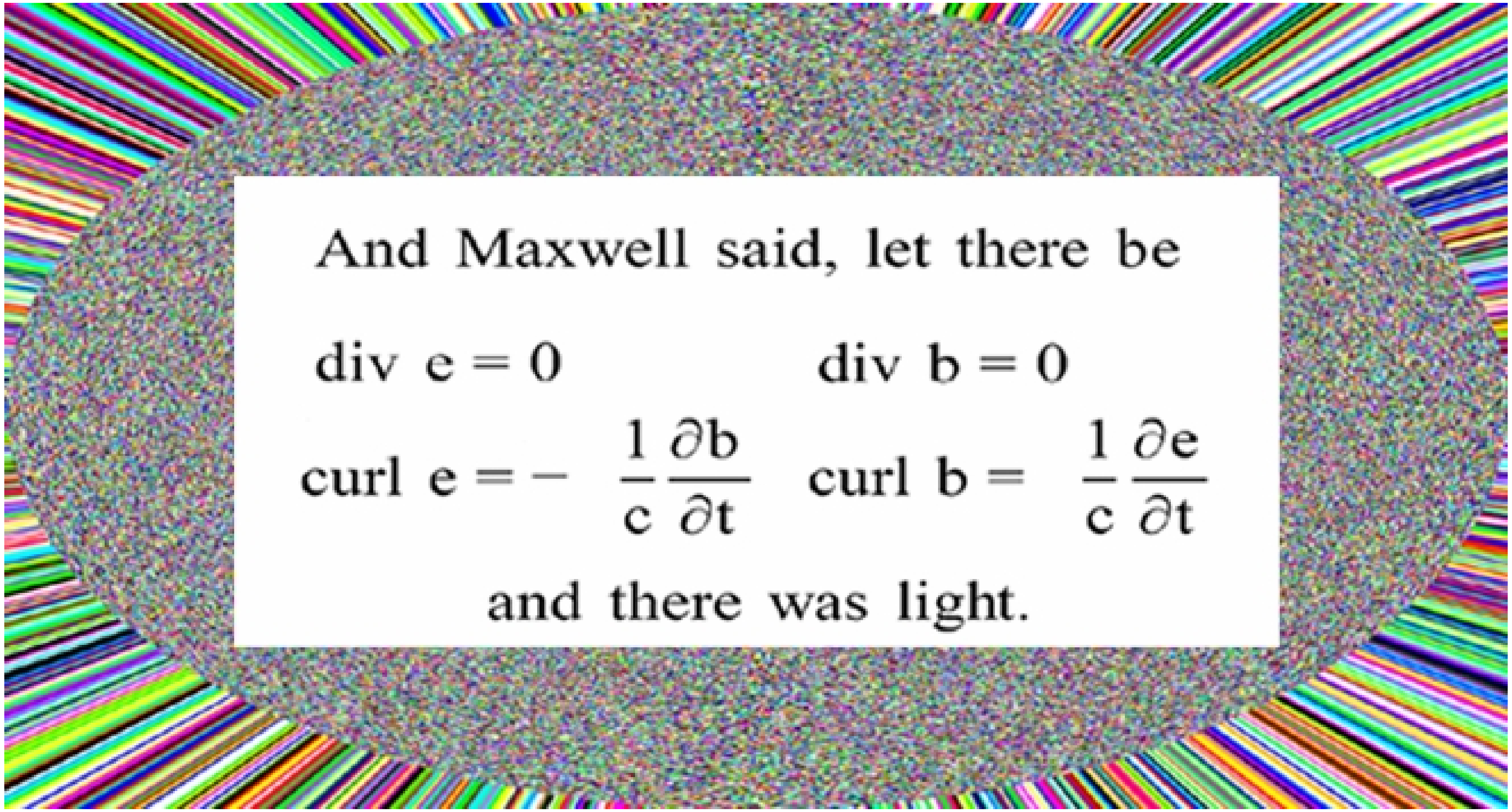




The Universe - Two Billion Years after the Big Bang
(Computer Animation - T. Theuns, MPA)

ESO PR Photo 19a/01 (18May 2001)

An ***Inverse Philosophy***



Let there be light and there was
Maxwell's Equations

Open (Inverse) Problem

GENERAL PHILOSOPHY

*Physics **is** the interaction (scattering) of waves with waves (no fields or particles)*

The inverse problem is to formally derive (at least) the following field equations:

Maxwell Equations	Einstein Equations	Dirac Equations

A Mathematical Information Theoretic Approach

Summary

- Low frequency scattering theory may provide an answer to the question of

  Why is an Einstein Ring Blue

- This observation leads to a new theory of gravity which is that gravity is a

  Low Frequency Scattering Effect

- Compatibility with GR is realised is the medium of propagation is

  Space-Time

Q & A