Quantum Electronics
Lecture 1

Introduction to Quantum Electronics

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Contents

♦ Course formalia
♦ Introduction to Quantum Electronics
♦ Résumé of Electromagnetic theory
♦ Optical coherence
Course literature

**Very helpful for the major part of the course topics**

*Book content:*


*Other materials for the topics not covered by the course book can be found in*

http://www.ict.kth.se/courses/IO2655/index.htm?links.html

Especially note **two online books** under: Electromagnetics, Optics and Photonic crystals
Credits and requirements

2 ECTS credits

To get the course approved you need to pass the exam

Examination – May 20

The written, close book exam will consist of questions or simple problems based on the lecture material

Formulas if needed will be provided with the exam sheet

50% right answers are required to pass
What is **Quantum Electronics, Photonics, and Optics**?

**Quantum Electronics**: “A loosely defined field concerned with the interaction of radiation and matter, particularly interactions involving quantum energy levels and resonance phenomena”

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**Quantum electronics** is approximately synonymous with

**Photonics (Optical Electronics)** – the science of generating, controlling, and detecting photons - optical equivalent of electronics

**Both include:**
emission, transmission, amplification, detection, modulation, and switching of light

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Classical optics is a sub-set of photonics. It covers part of light controlling

Modern optics is commonly categorized as photonics

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The theory of quantum optics provides an explanation of virtually all optical phenomena.

The electromagnetic theory of light (electromagnetic optics) provides the most complete treatment of light within the confines of classical optics.

Wave optics is a scalar approximation of electromagnetic optics.

Ray optics is the limit of wave optics when the wavelength is very short compared with structures.

Good link shortly reviewing Electromagnetics fundamentals: http://www.dur.ac.uk/g.h.cross/notes_b.pdf
Light Matter Interaction - levels of treatment

**Classical:** Lorentz dipole oscillator

**Semiclassical:** atoms quantized, light classical

**2nd quantization:** light and atoms quantized

**Full Quantum Electrodynamical** (QED)

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**Tab. 6.1** Treatment of light and matter by theoretical physics°.

<table>
<thead>
<tr>
<th></th>
<th>Matter</th>
<th>Light</th>
<th>Atomic motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical optics</td>
<td>C</td>
<td>C</td>
<td>C</td>
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<tr>
<td>Quantum electronics</td>
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<td>C</td>
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<tr>
<td>Quantum optics</td>
<td>Q</td>
<td>Q</td>
<td>C</td>
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<tr>
<td>Matter waves</td>
<td>Q</td>
<td>Q</td>
<td>Q</td>
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</table>

°C = classical physics; Q = quantum theory.
History of light and matter interactions

**Heinrich Hertz** 1857–1894

*discovered photoelectric effect*

**Lorentz** 1892 - **Classical electron theory**
- Nobel prize 1902 (with Zeeman)

Described the electromagnetic force acting on a charged particle
Atom - nucleus connected to electrons by a “spring”

**Thomson** 1897 – **”Discovered” electrons**
- Nobel prize 1906

Found that the cathode rays are deflected by an electric field and concluded that they were negatively charged particles
History of light and atom quantization (1)

Planck 1900 – **Quantized energy of atomic radiators**, explains black body radiation - Nobel prize 1918

Einstein 1905 - **Light quanta postulate** explains photoelectric effect - Nobel prize 1921

Bohr 1913 – **Quantized atom model**, explains spectral lines of hydrogen atom - Nobel prize 1922
History of light and atom quantization (2)

Einstein 1917 – **Quantified spont. emission, discovered stimulated emission**

Dirac 1927 - **Light-field quantization, relativistic description of electron** - *Nobel prize 1933 (with Schrödinger)*

*Paul Dirac 1902-1984*

Richard Feynman 1918–1988

Feynman, Dyson, Schwinger, Tonagawa 1940s - **Quantum electrodynamics** - *Nobel prize 1965*

Glauber 1963 – **Formulation of quantum theory to describe the detection process** - *Nobel prize 2005*
Nobel Prize in Physics  2005 for breakthroughs in modern optics

"for his contribution to the quantum theory of optical coherence"

"for their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique"

Roy J. Glauber  
1/2 of the prize  
USA

John L. Hall  
1/4 of the prize  
USA

Theodor W. Hänsch  
1/4 of the prize  
Germany

Showed how the quantum theory has to be formulated to describe the detection process
Applications of Optoelectronics

- Information, Communication

- Imaging

- Lighting and Displays

- Manufacturing and Quality

- Life Science and Health Care

- Safety and Security

http://www.photonics21.org/
High-capacity communications networks

Information-carrying capacity of light >> (10,000 times) than at radio frequencies

Think what this for instance means for the speed of your internet !!!

Or for the quality of image transfer
Basic fiber optic system

Transmitter - converts an electrical signal into a light signal
Optical fiber - carries the light
Receiver - accepts the light signal and converts it back into an electrical signal

Relies on: Low loss fiber transmission
Light generators and detectors
Opto-electronic interface / integration
Light guiding by internal reflection

Daniel Colladon’s Experiment (“light jet”) 1841

1950’s - first practical all-glass fiber developed and applied for image transmission (fiberscope) by Brien O’Brien at Narinder Kapany (USA)

He coined the term “fiber optics” - 1956

Cladding introduced by van Heel protected core surface from contamination and reduced losses

Still in 1960 very high transmission loss - 10 000 dB/km!
Towards optical communication

Breakthroughs:

1966 **High fiber loss attributed to impurities (not silica glass)**

Losses < 20 dB/km possible, Long-distance communications over single-mode fiber proposed (Standard Telecommunications Laboratories, UK - Kao and Hockham) *Research initially supervised by Antoni E. Karbowiak*

1970 **First fiber with loss < 20 dB/km at 633 nm (helium-neon) demonstrated** (Corning - Maurer, Keck, Schultz)

1970 **First continuous-wave room-temperature semiconductor lasers demonstrated** (Ioffe Physical Institute - Alferov's group, Bell Labs – Panish and Hayashi)

1978 **0.2 dB/km loss in single-mode fiber at 1.55 um (NTT) !!!**

1987 **First erbium-doped fiber amplifier for 1.55 um demonstrated** (Southampton University – Payne et al, AT&T Bell Laboratories – Desurvire et al)
2009 Nobel Prize in Physics for the masters of light

Charles K. Kao
1/2 of the prize
Standard Telecommunication Laboratories
Harlow, United Kingdom; Chinese University of Hong Kong
Hong Kong, China

Willard S. Boyle
1/4 of the prize
Bell Laboratories
Murray Hill, NJ, USA

George E. Smith
1/4 of the prize
Bell Laboratories
Murray Hill, NJ, USA

The physics behind IT - Nobel price in Physics 2000

Zhores I. Alferov, A.F. Ioffe Physico-Technical Institute, St. Petersburg, Russia.

Zhores I. Alferov and Herbert Kroemer receive the Nobel Prize for their work on semiconductor heterostructures used in high-speed electronics and optoelectronics.

Herbert Kroemer, University of California at Santa Barbara, USA.

Jack S. Kilby, Texas Instruments, Dallas, Texas, USA, receives the Nobel Prize for his part in the invention of the integrated circuit.

Invention of the electrical integrated circuit in 1958
Signal processing in electronic domain

O/E, E/O converters are bit rate and wavelength dependent. Bandwidth narrower than the optical one.
In 1960 Miller proposed integration of several optical components on one semiconductor chip and coined the term “integrated optics” - analogy to electronic integrated circuits.

The uses of photons instead of electrons would eliminate O/E converters - hence make the chip bit rate and wavelength transparent.

**AIM:** Light sources, modulators, switches, filters, splitters, waveguides, and detectors on a single integrated platform.
Signal processing in optical domain

Complex photonic Integrated circuit (PIC) is still a “holy grail” ...

**Drawbacks:**

- Big!! – at least 1000 times larger than its electronic counterpart
- High cost of developing new fabrication technology

"holy grail" - any ultimate, but elusive, goal pursued as in a quest

Święty Graal nadal pozostaje tajemnicą a legendy mówią, że aby doznać oświecenia należy poznać tajemnicę Graala

**The ongoing efforts to miniaturize PIC will be addressed in the next lectures**
Optoelectronic Integration
Maxwell equations

Ampère's circuital law (with Maxwell's correction)

\[ \nabla \times H = \frac{\partial D}{\partial t} + J \quad \nabla \cdot D = \rho \]
\[ \nabla \times E = -\frac{\partial B}{\partial t} \quad \nabla \cdot B = 0 \]

Gauss's law

\[ D = \varepsilon E, \quad B = \mu H \]

Maxwell's correction: electric field changing in time generates magnetic field

Faraday's law: a changing magnetic field induces an electric field

Self-sustaining "electromagnetic waves" can travel through empty space

http://www.plasma.uu.se/CED/Book/ - online book on "Electromagnetic Field Theory“ by Bo Thidé

For SI units see e.g. http://en.wikipedia.org/wiki/Maxwell's_equations
Electromagnetic Wave

Maxwell's hypothesis 1864: light is an electromagnetic wave

Electric and magnetic fields are oscillating perpendicularly to each other and to the direction of propagation
## Ranges of Electromagnetic Waves

**Optical communications**  
1260 - 1675 nm  
(near & short infrared)

<table>
<thead>
<tr>
<th>Name</th>
<th>Frequency</th>
<th>Wavelength ($\lambda$)</th>
<th>Time for one $\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extra Low Freq</td>
<td>60 Hz</td>
<td>5000 km ($5\times10^6$)</td>
<td>17 ms ($1.7\times10^{-2}$)</td>
</tr>
<tr>
<td>Audio Frequency</td>
<td>10 kHz ($1\times10^4$)</td>
<td>30 km ($3\times10^4$)</td>
<td>100 µs ($1\times10^{-4}$)</td>
</tr>
<tr>
<td>Radio Frequency</td>
<td>222 MHz ($2\times10^8$)</td>
<td>1.4 m</td>
<td>4.5 ns ($4.5\times10^{-9}$)</td>
</tr>
<tr>
<td>Microwave</td>
<td>10 GHz ($1\times10^{10}$)</td>
<td>30 mm ($3\times10^{-2}$)</td>
<td>100 ps ($1\times10^{-10}$)</td>
</tr>
<tr>
<td>Infrared (Heat)</td>
<td>10 THz ($1\times10^{13}$)</td>
<td>30 µm ($3\times10^{-5}$)</td>
<td>100 fs ($1\times10^{-13}$)</td>
</tr>
<tr>
<td>Visible</td>
<td>600 THz ($6\times10^{14}$)</td>
<td>500 nm ($5\times10^{-7}$)</td>
<td>1.7 fs ($1.7\times10^{-15}$)</td>
</tr>
<tr>
<td>Ultraviolet</td>
<td>$1\times10^{16}$ Hz</td>
<td>30 nm ($3\times10^{-8}$)</td>
<td>$1\times10^{-16}$ s</td>
</tr>
<tr>
<td>X-ray</td>
<td>$1\times10^{18}$ Hz</td>
<td>300 pm ($3\times10^{-10}$)</td>
<td>$1\times10^{-18}$ s</td>
</tr>
<tr>
<td>Gamma-ray</td>
<td>$1\times10^{20}$ Hz</td>
<td>3 pm ($3\times10^{-12}$)</td>
<td>$1\times10^{-20}$ s</td>
</tr>
</tbody>
</table>
Wave equation

\[ \nabla \times H = \frac{\partial D}{\partial t} + J \quad \nabla \cdot D = \rho \]
\[ \nabla \times E = -\frac{\partial B}{\partial t} \quad \nabla \cdot B = 0 \]

\[ \frac{J}{\rho} = 0 \]

\[ \nabla^2 E - \mu \varepsilon \frac{\partial^2 E}{\partial t^2} = -\nabla \left( \frac{1}{\varepsilon} E \cdot \nabla \varepsilon \right) \]
\( \varepsilon \) defines medium properties

When \( \varepsilon \) is constant, or varies slowly in comparison with the optical wavelength:

\[ \nabla^2 E - \mu \varepsilon \frac{\partial^2 E}{\partial t^2} = 0 \quad \text{Wave equation} \]

For monochromatic (single harmonic) fields:

\[ \nabla^2 E + k^2 E = 0 \quad \text{Helmholtz equation} \]

Plane wave:

\[ E = A \cos(kz - \omega t) \]

\[ k = \omega \sqrt{\mu \varepsilon} = \frac{\omega}{c} = \frac{2\pi}{\lambda} \]
Complex number convention

Complex number: \( W = Ae^{i\theta} = A[\cos(\theta) + i \sin(\theta)] \)

Plane wave: \( E = A \cos(kz - \omega t) \)

\[
E = \Re\left[Ae^{i(kz - \omega t)}\right] = \frac{1}{2} \left[Ae^{i(kz - \omega t)} + \text{c.c.}\right]
\]

For simplicity, \( \frac{1}{2} \Re \), or +c.c are commonly omitted: \( E = Ae^{i(kz - \omega t)} \)
Plane wave – complex amplitude

\[ \vec{E}(k,t) = \hat{i}_x E_x(k,t) + \hat{i}_y E_y(k,t) \]

\[ E_x = A_x e^{i(kz - \omega t + \varphi_x)} \]
\[ E_y = A_y e^{i(kz - \omega t + \varphi_y)} \]

\[ \vec{E} = \left[ \hat{i}_x A_x e^{i\varphi_x} + \hat{i}_y A_y e^{i\varphi_y} \right] e^{i(kz - \omega t)} \]
Light polarization

Polarization of a plane wave propagating in $z$ direction is defined as the curve traced in time in the $xy$ plane by the end point of the electric field vector.

**LINEAR:**

$$E(t) = \hat{x} E_0 \cos(\omega t)$$

**CIRCULAR:**

$$E(t) = \frac{E_0}{\sqrt{2}} [\hat{x} \cos(\omega t) + \hat{y} \sin(\omega t)]$$
Types of Polarization

In general, light is *elliptically polarized*

\[
\left( \frac{E_x}{A_x} \right)^2 + \left( \frac{E_y}{A_y} \right)^2 - \frac{2E_xE_y \cos \delta}{A_xA_y} = \sin^2 \delta
\]

\[\delta \equiv (\varphi_x - \varphi_y)\]

**Special cases:**

Zero phase difference (\(\delta = 0\)) gives oscillation along a line: *linear polarization*

Equal amplitudes (\(A_x = A_y\)) and \(\pi/2\) phase difference: *circular polarization*
Birefringence

\[ D = \varepsilon \cdot E \quad \text{and} \quad n^2 = \varepsilon \]

depends on the direction, so \( \varepsilon \) is a tensor

uniaxial crystal: \( n_x = n_y \equiv n_0 \neq n_z \equiv n_e \)

ordinary index (perpendicular to optic axis \( z \))

extraordinary index (along optic axis \( z \))

In the principal coordinate system off-diagonal elements vanish:

\[ D_x = \varepsilon_{11} E_x = n_0^2 E_x \qquad D_y = \varepsilon_{22} E_y = n_0^2 E_y \quad D_z = \varepsilon_{33} E_z = n_e^2 E_z \]

In general, directions of \( E \) and \( D \) are different!
Impermeability tensor – Index ellipsoid

\[ E = \varepsilon^{-1} D \]

Define: \[ \eta = \frac{1}{\varepsilon} \]

Impermeability tensor:

\[ \eta_{ij} = \frac{1}{n_{ij}^2} \]

Symmetric in lossless and optically inactive media:

\[ \eta_{ij} = \eta_{ji} \]

The index ellipsoid - convenient geometric representation

\[ \eta_{ij}x_i x_j = \sum_{ij} \eta_{ij} x_i x_j = 1 \]

in the principal coordinate system (crystal axes):

\[ \frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1 \]

where: \[ x_{11} \equiv x, \ x_{22} \equiv y, \ x_{33} \equiv z \]
Ordinary and extraordinary waves

In uniaxial crystals: \[ \frac{x_1^2}{n_0^2} + \frac{x_2^2}{n_0^2} + \frac{x_3^2}{n_e} = 1 \]

For any propagation direction \( k \) there are two allowed waves of two orthogonal polarizations:

- **Ordinary wave:** \( D_0 \) in the plane perpendicular to \( z \) for which \( n = n_0 \)
- **Extraordinary wave:** \( D_e \) perpendicular to \( D_0 \)
  
  \( n \) depends on the propagation direction:

\[
\frac{1}{n^2(\theta)} = \frac{\cos^2(\theta)}{n_0^2} + \frac{\sin^2(\theta)}{n_e^2} \quad \Rightarrow \quad n(0^\circ) = n_0 \quad n(90^\circ) = n_e
\]

Both \( D_0 \) and \( D_e \) are perpendicular to \( k \)
Normal surface or wavevector surface

For extraordinary wave $D$ is perpendicular to $k$, $E$ to $S$!
Positive and negative birefringence

Negative birefringence: 
\( n_1 = n_2 > n_3 \)

Positive birefringence: 
\( n_1 = n_2 < n_3 \)

\[ \frac{c}{n_e} \geq \frac{c}{n_o} \quad \text{o-wave travels slower} \]

\[ \frac{c}{n_e} \leq \frac{c}{n_o} \quad \text{e-wave travels slower} \]
Double refraction

**o-ray (ordinary)**

Obeys Snell’s Law and goes straight

Vibrates \perp plane containing ray and c-axis (“optic axis”)

**e-ray (extraordinary)**

Deflected

Vibrates \textbf{in} plane containing ray and c-axis

Double image:
Group velocity in a medium

\[ v_g \equiv \frac{d\omega}{dk} \]

\[ v_g \equiv \left[ \frac{dk}{d\omega} \right]^{-1} \]

Using \( k = \omega n(\omega) / c_0 \), calculate:

\[ \frac{dk}{d\omega} = \frac{n + \omega \frac{dn}{d\omega}}{c_0} \]

\[ v_g = \frac{c_0}{n + \omega \frac{dn}{d\omega}} = \frac{c_0}{n} \left( 1 + \frac{\omega}{n} \frac{dn}{d\omega} \right) \]

Group velocity = phase velocity \( (c_0/n) \), when \( \frac{dn}{d\omega} = 0 \), such as in vacuum.

Otherwise, except of regions close to material resonances, \( n \) increases with \( \omega \), \( \frac{dn}{d\omega} > 0 \), so

\[ v_g < \frac{c_0}{n} \]
Group velocity - normal dispersion regime

Normal material dispersion:

\[ n(\omega_{blue}) > n(\omega_{yellow}) > n(\omega_{red}) > 1, \quad \frac{dn}{d\omega} > 0 \]

\[ v_g = \frac{c_0}{n + \omega \frac{dn}{d\omega}} \]

\( v_g < c \)

\( v_g \) well characterizes velocity of energy carried by a pulse
“Group velocity” in anomalous dispersion regime

\[
\frac{dn}{d\omega} < 0 \rightarrow \quad V_g = \frac{c_o}{(n - \omega|dn/d\omega|)} > \frac{c_o}{n}
\]

Due to strong pulse distortions \(V_g\) loses the meaning of energy carried by a pulse, and can even be negative.

BUT… signal front velocity never exceeds \(c_o\) ! (in fact it is = \(c_o\))

Information cannot be sent faster than \(c_o\)

Read more about it in http://www.ict.kth.se/courses/IO2655/index.htm?links.html
Ubder: Electromagnetics, Optics
Chromatic and Group Velocity dispersion

\[ \varphi(\omega) = k(\omega) \, L \]

To account for dispersion, expand the phase in a Taylor series:

\[ k(\omega)L = k(\omega_0)L + k'(\omega_0)(\omega - \omega_0)L + \frac{1}{2} k''(\omega_0)(\omega - \omega_0)^2 L + ... \]

\[ k(\omega_0) = \frac{\omega_0}{v_\phi(\omega_0)} \quad k'(\omega_0) = \frac{1}{v_g(\omega_0)} \quad k''(\omega) = \frac{d}{d\omega} \left[ \frac{1}{v_g} \right] \]

**Phase velocity dispersion**

(variation in phase velocity with \( \omega \), separation of colors in a prism)

**Group velocity dispersion - GVD**

(variation in group velocity with \( \omega \), pulse broadening and "chirp")
Coherent and Random light

Temporal coherence

Spatial coherence
Coherence of waves

Waves are coherent when their relative phase is constant during the resolution time $\tau_D$ of the detector - temporal coherence, and within the resolution area $A_D$ - spatial coherence.

Coherence enables stationary (temporally and spatially constant) interference.
Temporal coherence is a measure of the correlation between the phases of a light wave at different points along the direction of propagation. Temporal coherence tells us how monochromatic a source is.

Perfectly coherent – unrealistic would require monochromatic waves lasting for ever

Partially coherent – realistic

Incoherent – ”white light”
Spatial coherence

Synchronized phases for rays emitted from different locations on the source during the temporal coherence time. *Spatial coherence tells us how uniform the phase of the wave front is*

The more extended the source the lower spatial coherence --> Point source would be ideal

Coherence degree - correlation between phase of the wave at two points

Often, achievable collimation degree is used for assessment of spatial coherence:

*The more collimated the beam the narrower its spectrum in wave vector space (flatter wave front), and the higher spatial coherence*

Incoherent beam - large uncertainty in relative phase
Spatial fringes - coherence area

A beam is temporally but not spatially, coherent:

Interference is incoherent (no fringes) far from the axis, where very different regions of the wave interfere.

Interference is coherent (sharp fringes) around the central axis, where same regions of the wave interfere.

Ac – Coherence area
Spatial filtering

The pinhole cleans up spatially incoherent wavefront. It produces a spatially coherent spherical wave (before lens) or spatially coherent plane wave (after lens).
Laser beam coherence

When the laser cavity has flat mirrors the beam is also highly **collimated**
Mutual (temporal) coherence

When two (or more) waves have the same frequency and their phase difference does not vary in time:

\[ E_1 = A_1 \exp[i(\omega t - k_1 \cdot r + \phi_1)] \]
\[ E_2 = A_2 \exp[i(\omega t - k_2 \cdot r + \phi_2)] \]

Their wave vectors must have the same length: \( |k_1| = |k_2| = k \), but not direction

Degree of coherence can be characterized by visibility of the interference pattern

\[ I = |E_1 + E_2|^2 = I_1 + I_2 + 2A_1 \cdot A_2 \cos(K \cdot r - \phi) = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(K \cdot r - \phi) \]

\[ I_i = |E_i|^2, \ K = k_1 - k_2, \text{ and } \phi = \phi_1 - \phi_2. \]

\[ \Lambda = \frac{2\pi}{|K|} = \frac{\lambda}{2 \sin(\theta/2)} \]

Period of the interference fringes
Degree of mutual coherence

\[ \gamma_{12} = \left( \frac{\langle \mathbf{E}_1^* \cdot \mathbf{E}_2 \rangle}{\left( \langle \mathbf{E}_1^* \cdot \mathbf{E}_1 \rangle \right)^{1/2} \left( \langle \mathbf{E}_2^* \cdot \mathbf{E}_2 \rangle \right)^{1/2}} \right)_{r=0} = \langle e^{i(\phi_2 - \phi_1)} \rangle = \frac{1}{\tau_D} \int_0^{\tau_D} e^{i(\phi_2 - \phi_1)} \, dt \]

\( \tau_D \) - detector time constant
\( \gamma_{12} = |\gamma_{12}| \exp(i\alpha) \)

If \( \phi_1 - \phi_2 \) varies in time the interference pattern time-averaged by detector "smears"

\[ \langle \langle I \rangle \rangle = |\mathbf{E}_1 + \mathbf{E}_2|^2 = I_1 + I_2 + 2\mathbf{A}_1 \cdot \mathbf{A}_2 |\gamma_{12}| \cos(\mathbf{K} \cdot \mathbf{r} - \alpha) \]

Modulation depth

Complete mutual coherence: \( |\gamma_{12}| = 1 \)
Partial coherence: \( 0 < |\gamma_{12}| < 1 \)
Complete mutual incoherence: \( |\gamma_{12}| = 0 \)
Temporal self-coherence

Coherence of two parts of the same wave

Temporal coherence can be measured in a Michelson interferometer. The wave is combined with a copy of itself that is delayed by time $\tau$ by moving the object mirror.

Coherence time: $\tau_c = \int_{-\infty}^{\infty} |\gamma(\tau)|^2 d\tau$

The longest time delay for which the phases are correlated (fringes are visible): $\tau_c \rightarrow O(1/\Delta \nu)$.

A rule of thumb is that 

Coherence length: $l_c = c\tau_c$ (more practical in the lab)

The longest propagation length over which coherence is preserved.

For LED $l_c$ is of the order of microns, for a laser diode – centimeters, for a gas laser – meters!
Degree of temporal self-coherence – formulae

Similar to those for mutual coherence, but here time variation of both intensity and the phase are included by introducing time dependent complex amplitude \( A(t) \), so that one can also analyze pulses

Delayed parts of the same beam:

\[
E_1 = A(t) \exp[i(\omega t - k_1 \cdot r)]
\]

\[
E_2 = A(t + \tau) \exp[i(\omega t + \omega \tau - k_2 \cdot r)]
\]

\[
\gamma(\tau) = \frac{\langle\langle E^*(t) E(t + \tau) \rangle\rangle}{\langle\langle E^*(t) E(t) \rangle\rangle^{1/2} \langle\langle E^*(t + \tau) E(t + \tau) \rangle\rangle^{1/2}}
\]

\[
\gamma(\tau) = \frac{\langle\langle E^*(t) E(t + \tau) \rangle\rangle}{\langle\langle E^*(t) E(t) \rangle\rangle}
\]

\( A(t) \) – slowly varying amplitude, e.g. envelope of a pulse at the central frequency \( \omega \)

Average correlation between field value at any pair of times, separated by delay \( \tau \)